Set 2: Multiple-Choice Questions on Limits and Continuity

1. $\lim_{x\to 2}\frac{x^2-4}{x^2+4}$ is

   (A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) ∞

2. $\lim_{x\to\infty}\frac{4-x^2}{x^2-1}$ is

   (A) 1 (B) 0 (C) -4 (D) -1 (E) ∞

3. $\lim_{x\to 3}\frac{x-3}{x^2-2x-3}$ is

   (A) 0 (B) 1 (C) $\frac{1}{4}$ (D) ∞ (E) none of these

4. $\lim_{x\to 0}\frac{x}{x}$ is

   (A) 1 (B) 0 (C) ∞ (D) -1 (E) nonexistent

5. $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$ is

   (A) 4 (B) 0 (C) 1 (D) 3 (E) ∞

6. $\lim_{x\to\infty}\frac{4-x^2}{4x^2-x-2}$ is

   (A) -2 (B) $-\frac{1}{4}$ (C) 1 (D) 2 (E) nonexistent

7. $\lim_{x\to-\infty}\frac{5x^3+27}{20x^2+10x+9}$ is

   (A) $-\infty$ (B) -1 (C) 0 (D) 3 (E) ∞

8. $\lim_{x\to\infty}\frac{3x^2+27}{x^3-27}$ is

   (A) 3 (B) ∞ (C) 1 (D) -1 (E) 0

18. $\lim_{x\to 0} \sin\frac{1}{x}$ is

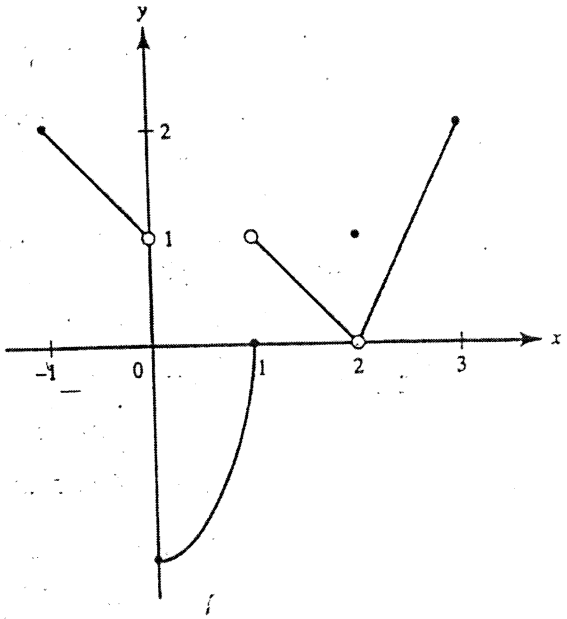
(A) ∞ (B) 1 (C) nonexistent (D) -1 (E) none of these

19. Which statement below is true about the curve $y = \dfrac{2x^2+4}{2+7x-4x^2}$?

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote
(B) The line $x = 1$ is a vertical asymptote
(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote
(D) The graph has no vertical or horizontal asymptotes
(E) The line $y = 2$ is a horizontal asymptote

Questions 20 through 24 are about the function f shown in the graph and defined below:

$$f(x) = \begin{cases} 1-x & -1 \le x < 0 \\ 2x^2-2 & 0 \le x \le 1 \\ -x+2 & 1 < x < 2 \\ 1 & x = 2 \\ 2x-4 & 2 < x \le 3 \end{cases}$$



20. $\lim_{x\to 2} f(x)$

(A) equals 0 (B) equals 1 (C) equals 2
(D) does not exist (E) none of these

21. The function f is defined on $[-1,3]$

(A) if $x \neq 0$ (B) if $x \neq 1$ (C) if $x \neq 2$
(D) if $x \neq 3$ (E) at each x in $[-1,3]$

22. The function has a removable discontinuity at

(A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) $x = 3$ (E) none of these

23. On which of the following intervals is f continuous?

(A) $-1 \le x \le 0$ (B) $0 < x < 1$ (C) $1 \le x \le 2$
(D) $2 \le x \le 3$ (E) none of these

24. The function has a jump discontinuity at

(A) $x = -1$ (B) $x = 1$ (C) $x = 2$
(D) $x = 3$ (E) none of these

25. $\lim_{x \to \infty} \frac{2x^2+1}{(2-x)(2+x)}$ is

(A) -4 (B) -2 (C) 1 (D) 2 (E) nonexistent

26. $\lim_{x \to 0} \frac{|x|}{x}$ is

(A) 0 (B) nonexistent (C) 1 (D) -1 (E) none of these

27. $\lim_{x \to \infty} x \sin \frac{1}{x}$ is

(A) 0 (B) ∞ (C) nonexistent (D) -1 (E) 1

28. $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x}$ is

(A) 1 (B) 0 (C) ∞ (D) nonexistent (E) none of these

29. Let $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases}$

Which of the following statements, I, II, and III, are true?

I. $\lim_{x \to 1} f(x)$ exists II. $f(1)$ exists III. f is continuous at $x = 1$

(A) only I (B) only II (C) I and II
(D) none of them (E) all of them

30. If $\begin{cases} f(x) = \frac{x^2-x}{2x} \text{ for } x \neq 0, \\ f(0) = k, \end{cases}$
and if f is continuous at $x = 0$, then $k =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

31. Suppose $\begin{cases} f(x) = \frac{3x(x-1)}{x^2-3x+2} \text{ for } x \neq 1, 2, \\ f(1) = -3, \\ f(2) = 4. \end{cases}$

Then $f(x)$ is continuous

(A) except at $x = 1$ (B) except at $x = 2$ (C) except at $x = 1$ or 2
(D) except at $x = 0$, 1, or 2 (E) at each real number

32. The graph of $f(x)=\frac{4}{x^2-1}$ has

(A) one vertical asymptote, at $x = 1$.
(B) the y-axis as vertical asymptote.
(C) the x-axis as horizontal asymptote and $x = \pm 1$ as vertical asymptotes.
(D) two vertical asymptotes, at $x = \pm 1$, but no horizontal asymptote.
(E) no asymptote.

33. Suppose $\lim_{x\to -3^-} f(x)=-1$; $\lim_{x\to -3^+} f(x)=-1$; $f(-3)$ is not defined. Which, if any, of the following statements is not necessarily true.

(A) $\lim_{x\to -3} f(x)=-1$.
(B) f has a removable discontinuity at $x = -3$.
(C) If we redefine $f(-3)$ to be equal to -1, then the new function will be continuous at $x = -3$.
(D) f is continuous everywhere except at $x = -3$.
(E) All of the preceding statements must be true.

34. The graph of $y=\frac{2x^2+2x+3}{4x^2-4x}$ has

(A) a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptotes.
(B) no horizontal asymptotes but two vertical asymptotes, at $x = 0$ and $x = 1$.
(C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$.
(D) a horizontal asymptote at $x = 2$ but no vertical asymptotes.
(E) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$.

35. Let $f(x)=\begin{cases}\frac{x^2+x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$

Which of the following statements, I, II, and III, are true?

I. $f(0)$ exists II. $\lim_{x\to 0} f(x)$ exists III. f is continuous at $x = 0$

(A) only I (B) only II (C) I and II only
(D) all of them (E) none of them