Definite Integral ↔ Limit of a Riemann Sum

The definition of a **definite integral** is $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x_k$.

Concept: $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left(f(a+k\Delta x)\Delta x\right) = \lim_{n\to\infty}\sum_{k=1}^{n}\left(\underbrace{f\left(a+k\frac{b-a}{n}\right)}_{\text{using right-hand heights}}\overbrace{\frac{b-a}{n}}^{\text{width}}\right)$

where n is the number of subdivisions.

Example 1 Write the following limit as a definite integral. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{3+\frac{2k}{n}}\bullet\frac{2}{n}\right)$	
1. Determine Δx	
2. Determine a	
3. Determine b	
5. Rewrite the integral	

Practice

Write each Riemann Sum as a definite integral.
Do <u>not</u> evaluate.

1. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(f\left(2+k\bullet\frac{3}{n}\right)\frac{3}{n}\right)$

2. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\cos\left(0+\frac{k\pi}{n}\right)\frac{\pi}{n}\right)$

3. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt[3]{2+\frac{5k}{n}}\bullet\frac{5}{n}\right)$

4. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(3\left(\frac{k}{n}+4\right)+2\right)\frac{1}{n}\right)$

5. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\cos\left(\frac{k\pi}{3n}\right)\right)\frac{\pi}{3n}\right)$