Area under the Curve

To approximate the area between a function and the *x*-axis, create rectangles that best fit and find the sum of the area of these rectangles. A curve above the *x*-axis is said to enclose “positive” area while a curve below the *x*-axis encloses “negative” area.



Riemann Sum: LRAM, MRAM, RRAM

Riemann Sum uses regular intervals when creating rectangles to approximate the area under the curve, *Rectangular Approximation Method* (RAM). LRAM creates rectangles with the height determined by the upper left-hand corner intersecting with the function. In MRAM, the function intersects at the upper middle part of the rectangle. In RRAM, the height of each rectangle is determined by the upper right-hand corner. LRAM, MRAM, and RRAM can be seen in that respective order below.



Trapezoidal Rule

Trapezoidal rule uses trapezoids to approximate the area between a function and the *x*-axis. The height, *h*, of each trapezoid is the constant interval and the bases are the *y* values at each interval.



Definite Integral



As the width of each rectangle decreases, the accuracy of the area approximation increases. Creating more rectangles over an interval yields a better answer. Thus, breaking an interval into infinitely many rectangles yields an integral from *a* to *b*. *dx* keeps track of variable of integration and comparable to .

Indefinite Integrals

Antidifferentiation is the inverse of differentiation: an antiderivative of is any function whose derivative is equal to : . There are many functions with the same derivative; these functions usually differ by a constant *C*. Therefore, the family of antiderivatives of are . Thus, the indefinite integral of :

Fundamental Theorem of Calculus

Part 1

If *f* is continuous on , then the function

Has a derivative at every point *x* in , and

Part 2 (*Integral Evaluation Theorem*)

If *f* is continuous at every point of , and if *F* is an antiderivative of *f* on , then

Integral Properties

 and are separate functions

Constant Multiple: *k* is any real number

Additivity: for any *b*, such that

Order of Integration

Zero Integral

Domination:

Basic Integrals

*u*-Substitution

This is the chain rule for integrals and it is used for composite functions and products. Let , then

or, with limits,

Example:

Piecewise Integration

If , then the integral of from *a* to *c* is

Absolute Value Integration

Rewrite as *piecewise function*. Solve for *x* inside the absolute value symbol. The inside of the absolute value should be positive with and negative with . Then, integrate as a piecewise function.

If , then the integral of from *a* to *c* is

Original Equation

Given a derivative and an initial value, to find the original equation:

1. Take the antiderivative.
2. Substitute the initial value.
3. Solve for the constant, *C*.
4. Rewrite equation.

Examples

Find the particular solution to the equation whose graph passes through the point (1, 0).

Apply the initial condition (1, 0).

Solve for *C*.

Rewrite Equation.

Find the particular solution to the equation whose graph passes through the point (0, 3).

Apply the initial condition (0, 3).

Solve for *C*.

Rewrite Equation.

Physical Motion

Displacement: s

Velocity: v

Acceleration:

Conversely,

Total distance traveled from to is

Example

A particle travels with velocity m/sec for sec. What is the particle’s displacement? What is the total distance traveled?

Consumption over Time

Let be a rate at which something in consumed over a period from to Then, the total consumption can be modeled by the integral

Hooke’s Law

Hooke’s Law for springs says that the force it takes to stretch or compress a spring *x* units from its natural (unstressed) length is a constant times *x*: . The amount of Work needed to stretch or compress a spring *x* units from its natural (unstressed) length can be found by solving the integral below.

Area

If *f* and *g* are continuous with throughout , then the area between the curves from *a* to *b* is the integral of from *a* to *b*,

Example

Find the area of the region enclosed by the parabola and the line .

The limit of integration are found by finding where the two equations intersect .

Thus, the area of the region is 4.5 square units.

Volume

If a solid in space is oriented so that the area of a cross-section perpendicular to the *x*-axis is given by . Then, the volume of solid bounded by the planes and is

Disk Method

Volume of a solid that revolves around the *x*-axis bounded by and is

Shell Method

Volume of a solid that revolves around the *y*-axis bounded by and is

Washer Method

If between a and b, the volume of a solid that revolves around the *x*-axis bounded by and is

Mean (Average) Value Theorem for Integrals

If *f* is continuous on , then at some point *c* in ,

The Mean Value Theorem for Integrals is the ***average value***of a function over an interval. This is not to be confused by the Mean Value Theorem for Derivatives. The MVT for Derivatives is the ***average rate of change*** of a function over an interval.

Example

What is the average value of the cosine function on the interval ?

The average value of cosine is –0.450.

If the average value of the function *f* on the interval is 10, then ?

 is equal to .

Separable Differential Equations

A differential equation expressed in the form is called separable, if it is possible to separate the variables completely. To integrate separable differential, separate the variables and rewrite in the form

Then integrate each side separately. It is sufficient to write only one constant, *C*.

Example

Exponential Growth and Decay

Law of Exponential Change

If *y* changes at a rate proportional to the amount present and is the initial amount at , then . The constant *k* is the growth constant if or the decay constant if .

Half-Life

The half-life, *h*, of a radioactive substance with rate constant *k* is .

Modeling Growth with other Bases

 where the base *b* is the multiplied by over the time period .

Example

Scientists who use carbon-14 dating use 5700 years for its half-life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

The sample is about 866 years old.

Slope Fields

A slope field graphically shows the family of solution to a differentiable equation in form. To construct a slope field, for each point , draw a short line segment with slow . To interpret a slope field (or find a particular solution) at a given point, follow the shape of the slope field through that point.



1. A slope field for the differential equation
2. The same slope field with the graph of the solution through superimposed.