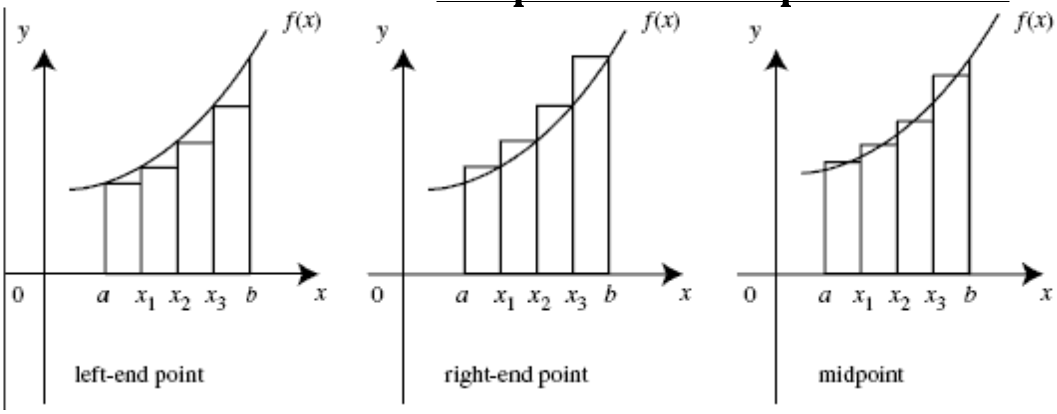
Lesson #79

Midpoint and Trapezoid Rule



Example #1:
Let $f(x) = e^x$.
Set up an approximation for the area under the curve of $f(x)$ from x=1 to x=3 using a left Riemann sum with 4 equal subintervals.

Set up an approximation for $\int_1^3 f(x)\,dx$ using a right Riemann sum with 4 subintervals of equal length.

Use a midpoint Riemann sum with 4 equal subintervals to approximate $\int_1^3 f(x)\,dx$.

Example #2:
A rocket has positive velocity v(t) after being launched upward. The velocity of the rocket is recorded for select values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Write an integral expression in terms of v(t) for the average velocity of the rocket from t=10 seconds to t=70 seconds. Estimate the average velocity of the rocket from t=10 seconds to t=70 seconds using a midpoint Riemann sum with 3 subintervals of equal length.

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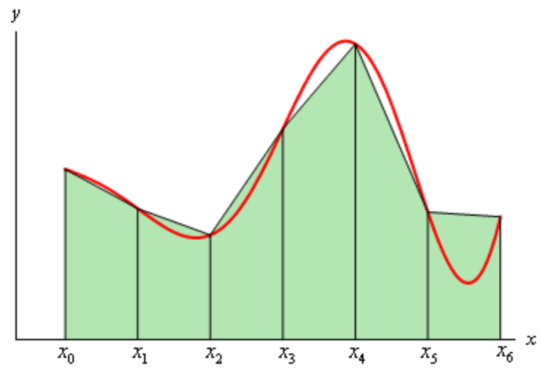
Trapezoid Rule:



Area of 1 Trapezoid:

If I have several trapezoids in a row with the same width…

Trapezoid Rule when the <u>subintervals are equal</u>:

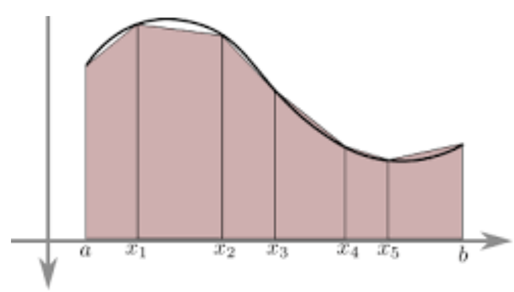
Formula: Area $\approx \frac{1}{2}\frac{b-a}{n}[y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n]$

where n is the number of subintervals.

<u>Example #3:</u> Use trapezoid rule to find the area from x=-3 to x=0 of $f(x) = 5x^2\sin(e^x)$ using 3 subintervals of equal length.

Trapezoid Rule when the <u>subintervals are unequal</u>:

You have to calculate the area of each trapezoid separately and then add the areas together.



<u>Example #4:</u>

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity, v, measured in feet per second is shown in the table above.

Using appropriate units, explain the meaning of $\int_{25}^{50} v(t)dt$ in terms of the car's motion.

Approximate $\int_{25}^{50} v(t)dt$ using a trapezoidal approximation with the three subintervals determined by the table.