

Inverse Trig Functions

Recall that for a function f and its inverse g , we have:

$$f(g(x)) = g(f(x)) = x.$$

Example #1:

Prove that $f(x) = \sqrt[3]{x-2}$ and $g(x) = x^3 + 2$ are inverses.

A function has an inverse if it passes the horizontal line test.

Example #2: $y = \sin x$

Now restrict $y = \sin x$ to $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Standard Restricted Domains		
Function	Domain	Range
$\sin^{-1}(x)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}(x)$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}(x)$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\cot^{-1}(x)$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}(x)$	$(-\infty, -1] \cup [1, \infty)$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
$\csc^{-1}(x)$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$

Example #3: Find the value of $\cos^{-1}\left(\frac{1}{2}\right)$.

** Think of this as “Where is cosine equal to $\frac{1}{2}$?”

Example #4: Find the value of $\cot^{-1}(-1)$.

Example #5: Find the value of $\sec^{-1}(-2)$.

Example #6: Find the **exact** value of $\cos\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$

Example #7: Find the **exact** value of $\cos^{-1}\left(\cos\left(\frac{12\pi}{7}\right)\right)$.