

## Differential Equations

- An equation involving one or more derivatives
- A first order DE has a first derivative in it.
- A second order DE has a second derivative in it.

We will use the “separation of variables” method:

Example #1:  $\frac{dy}{dx} = x^3$

Step 1: Put x's with x's on one side.

Step 2: Put y's with y's on the other side.

Step 3: Integrate both sides.

Step 4: Solve for y, if possible.

\*\* Don't forget the + C!\*\*

Example #2:

Solve using separation of variables.

$$\frac{dy}{dx} = -4xy^2$$

Lesson #70/71

Example #3:

$$x(y - 1) \frac{dy}{dx} = y$$

Example #4:

$$\frac{dy}{dx} = xe^y$$

But now, we want to find the solution that passes through the point (2,0).

**Solving Separable DE with  
Initial Conditions**

- 1- Separate the variables
- 2- Integrate both sides
- 3- Don't forget +C
- 4- Plug in initial condition to solve for C
- 5- Plug c into the equation you used to solve, then isolate y

Example #5: Find the equation of the solution curve that passes through the point (1, -1) for the DE.

$$\frac{dy}{dx} = \frac{y^2}{3\sqrt{x}}$$

**Growth and Decay Formula:**

The rate of change of  $y(t)$  is directly proportional to the amount present,  $y$ .

$$\frac{dy}{dt} = ky$$

**Solve by separation of variables:**

Lesson #70/71

**Separable Differential Equations with Log**

2004 Question 5c)

Consider the differential equation  $\frac{dy}{dx} = x^4(y - 2)$ .

Find the particular solution of  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 0$ .

\*To figure out what to pull out of the absolute value, we plug in the \_\_\_\_\_. If the number is positive, we \_\_\_\_\_. If the number is negative, we \_\_\_\_\_.

2006 Question 5b)

Consider the differential equation  $\frac{dy}{dx} = \frac{1+y}{x}$ , where  $x \neq 0$ . Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(-1) = 1$ .