Lesson #48 & 49 & 50

Area as Limits – Riemann Sum

Definition:

The area of a region R is the limit of simple regions whose areas are known.

$$Area = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

** Think of it as length times width of a rectangle

$$\Delta x = \underline{\qquad\qquad}$$

Example 1:

a) Approximate the area under the curve $f(x) = x^2 + 2,\ -2 \leq x \leq 1$ with a Riemann Sum, using 3 sub-intervals and right endpoints.

b) Approximate the area under the curve $f(x) = x^2 + 2,\ -2 \leq x \leq 1$ with a Riemann Sum, using 6 sub-intervals and right endpoints.

c) Why is the area we found in b) more accurate then the area we found in a)? What would happen if we increased the number of sub-intervals? If n in the formula above represents the number of rectangles we are using in our sum, why would we get an exact calculation for area as the limit of n approaches infinity?

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Example 2:

Approximate the area under the curve $f(x) = \sqrt{x+1}, -1 \leq x \leq 0$ with a Riemann sum, using four sub-intervals and left end points.

Is this approximation an underestimate or an over estimate? Why?

You Try:

Approximate the area under the curve $f(x) = x^3 - 3,\ 1 \leq x \leq 3$

a) with a Riemann sum, using three sub-intervals and right end points.

b) using five sub-intervals and left end points.

What you just calculated above is an approximation for $\int_1^3 x^3 - 3dx$. This means that the graphical representation of an integral is ____.

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Graphical Representation of the Derivative:
Graphical Representation of the Integral:

Area with Integrals

Sketch the region whose area is represented by the definite integral. Then calculate the exact area using geometric formulas.

1. $\int_0^2 \left(1 - \frac{1}{2}x\right) dx$

2. $\int_2^3 \left(1 - \frac{1}{2}x\right) dx$

3. $\int_0^3 |x - 2|\, dx$

4. $\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \sin(x)\, dx$