

Things to Help with L#28 HW

11. $x^2 + y^2 = 100$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

13. $x^2y + 3xy^3 - x = 3$

$$\boxed{\frac{dy}{dx} = \frac{1 - 2xy - 3y^3}{x^2 + 9xy^2}}$$

15. $x^{-1/2} + y^{-1/2} = 1$

$$\begin{aligned} \frac{dy}{dx} &= \frac{-x^{-3/2}}{y^{-3/2}} \\ &= \boxed{\frac{-y^{3/2}}{x^{3/2}}} \end{aligned}$$

17. $\sin(x^2y^2) = x$

$$\begin{aligned} \cos(x^2y^2)(2xy^2 + x^2(2y) \cdot \frac{dy}{dx}) &= 1 \\ 2xy^2 \cos(x^2y^2) + 2x^2y \cos(x^2y^2) \cdot \frac{dy}{dx} &= 1 \end{aligned}$$

$$2x^2y \cos(x^2y^2) \frac{dy}{dx} = 1 - 2xy^2 \cos(x^2y^2)$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 2xy^2 \cos(x^2y^2)}{2x^2y \cos(x^2y^2)}}$$

19. $\tan^3(xy^2 + y) = x$

$$3 \tan^2(xy^2 + y) \sec^2(xy^2 + y) (y^2 + 2xy \cdot \frac{dy}{dx} + \frac{dy}{dx}) = 1$$

$$21. 2x^2 - 3y^2 = 4$$

$$\frac{dy}{dx} = \frac{2x}{3y}$$

$$\frac{d^2y}{dx^2} = \frac{6y - 6x \cdot \frac{dy}{dx}}{9y^2}$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{6y - 12x^2}{9y^2} \\ &= \frac{6y^2 - 4x^2}{9y^3} \\ &= \frac{-2(2x^2 - 3y^2)}{9y^3} \\ &= \frac{-2(4)}{9y^3} = \boxed{\frac{-8}{9y^3}}\end{aligned}$$

$$25. y + \sin y = x$$

$$\frac{dy}{dx} = \frac{1}{1 + \cos y} = (1 + \cos y)^{-1}$$

$$\frac{d^2y}{dx^2} = \frac{\sin y}{(1 + \cos y)^2} \cdot \frac{dy}{dx}$$

$$= \boxed{\frac{\sin y}{(1 + \cos y)^3}}$$

$$23. x^3y^3 - 4 = 0$$

$$\frac{dy}{dx} = \frac{-y}{x} \quad \text{minus } -y$$

$$\frac{d^2y}{dx^2} = \frac{x \left(-\frac{dy}{dx} \right) + y}{x^2}$$

$$= \frac{x \left(\frac{y}{x} \right) + y}{x^2} = \boxed{\frac{2y}{x^2}}$$

$$27. x^2 + y^2 = 1$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

↓

the same as

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$$

when

$$y = \sqrt{1-x^2}$$

$$29. x^4 + y^4 = 16$$

$$\frac{dy}{dx} = \frac{-x^3}{y^3}$$

$$\text{pt. } (1, \sqrt[4]{15})$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{(\sqrt[4]{15})^3}}$$

Lesson #28, prob #31

$$2(x^2 + y^2)^2 = 25(x^2 - y^2)$$

$$2(x^4 + 2x^2y^2 + y^4)$$

$$2x^4 + 4x^2y^2 + 2y^4 = 25x^2 - 25y^2$$

$$8x^3 + 8xy^2 + 8x^2y \cdot \frac{dy}{dx} + 8y^3 \cdot \frac{dy}{dx} = 50x - 50y \cdot \frac{dy}{dx}$$

$$50y \cdot \frac{dy}{dx} + 8x^2y \cdot \frac{dy}{dx} + 8y^3 \cdot \frac{dy}{dx} = -8x^3 - 8xy^2 + 50x$$

$$= \frac{-8x^3 - 8xy^2 + 50x}{50y + 8x^2y + 8y}$$

(3, 1)

$$\Rightarrow \frac{-8(27) - 8(3)(1) + 50(3)}{50 + 8(9)(1) + 8}$$

$$= \frac{-90}{130} = \boxed{\frac{-9}{13}}$$