Name: _____ **Class:** _____ **Date:** _____ **ID: A**

2013-2014 AP Calculus AB Unit 3 Assessment

Multiple Choice
Identify the choice that best completes the statement or answers the question.

A calculator may NOT be used on this part of the exam. (30 minutes)

1. If $y = \sin^{-1} x - \sqrt{1-x^2}$, then $\frac{dy}{dx} =$

   a) $\frac{1}{2\sqrt{1-x^2}}$

   b) $\frac{2}{\sqrt{1-x^2}}$

   c) $\frac{1+x}{\sqrt{1-x^2}}$

   d) $\frac{x^2}{\sqrt{1-x^2}}$

   e) $\frac{1}{\sqrt{1+x}}$

2. A balloon is being filled with helium at the rate of 4 ft^3/min. The rate, in square feet per minute, at which the surface area is increasing when the volume $\frac{32\pi}{3}$ ft^3 is

   a) 4π

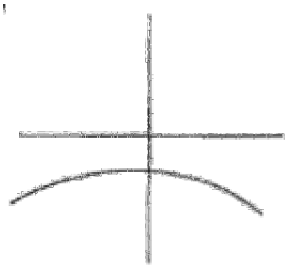
   b) 2

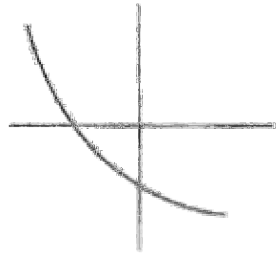
   c) 4

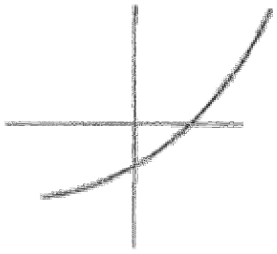
   d) 1

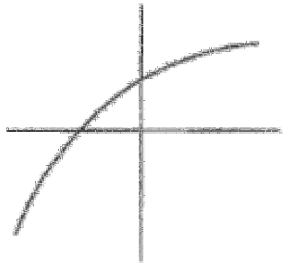
   e) 2π

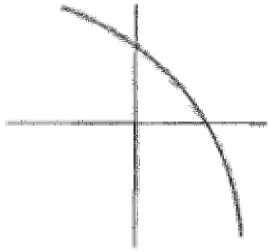
3. For which curve shown below are both f' and f'' negative?

a) 

b) 

c) 

d) 

e) 

4. The position of a particle moving along a straight line is given by $s = t^3 - 6t^2 + 12t - 8$. The distance s is increasing for

a) $t < 2$

b) all t except $t = 2$

c) $1 < t < 3$

d) $t < 1$ or $t > 3$

e) $t > 2$

5. The best approximation, in cubic inches, to the increase in volume of a sphere when the radius is increased from 3 to 3.1 inches is

   a) $\frac{0.04\pi}{3}$

   b) 0.04π

   c) 1.2π

   d) 3.6π

   e) 36π

6. When $x = 3$, the equation $2x^2 - y^3 = 10$ has the solution $y = 2$. When $x = 3.04$, $y \approx$

   a) 1.6

   b) 1.96

   c) 2.04

   d) 2.14

   e) 2.4

7. $\lim_{x \to \infty} \frac{2x^2 + 1}{(2-x)(2+x)}$ is

   a) –4

   b) –2

   c) 1

   d) 2

   e) nonexistent

8. The height of a rectangular box is 10 in. Its length increases at the rate of 2 in./sec; its width decreases at the rate of 4 in./sec. When the length is 8 in and the width is 6 in., the rate, in cubic inches per second, at which the volume of the box is changing is

   a) 200

   b) 80

   c) –80

   d) –200

   e) –20

9. The equation of the tangent to the curve $y = x\sin x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

   a) $y = x - \pi$

   b) $y = \frac{\pi}{2}$

   c) $y = \pi - x$

   d) $y = x + \frac{\pi}{2}$

   e) $y = x$

10. Suppose $f(x) = \begin{cases} \frac{3x(x-1)}{x^2 - 3x + 2} & x \neq 1, 2 \\ -3 & x = 1 \\ 4 & x = 2 \end{cases}$. Then $f(x)$ is continuous

    a) except at $x = 1$

    b) except at $x = 2$

    c) except at $x = 1, 2$

    d) except at $x = 0, 1, 2$

    e) at all real numbers

11. The first-quadrant point on the curve $y^2 x = 18$ that is closest to the point $(2, 0)$ is

    a) $(2, 3)$

    b) $(6, \sqrt{3})$

    c) $(3, \sqrt{6})$

    d) $(1, 3\sqrt{2})$

    e) none of these

12. The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is

    a) 0
    b) 2
    c) 6
    d) –2
    e) none of these

13. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

    a) $f(0)$ is not defined
    b) $f(x)$ is not continuous on $[-8, 8]$
    c) $f'(-1)$ does not exist
    d) $f(x)$ is not defined for $x < 0$
    e) $f'(0)$ does not exist

14. The sum of the squares of two positive number is 200; their minimum product is

    a) 100
    b) $25\sqrt{7}$
    c) 28
    d) $24\sqrt{14}$
    e) none of these

15. What is the value of $\lim_{x \to 1} f(x)$?

$$f(x) = \begin{cases} 2 - x, & x \le 1 \\ \frac{x}{2} + 1, & x > 1 \end{cases}$$

    a) 2.5
    b) 1.5
    c) 1
    d) 0
    e) does not exist

A graphing calculator is REQUIRED for some questions on this part of the exam. (27 minutes)

16. The point on the curve $y = \sqrt{2x+1}$ at which the normal is parallel to the line $y = -3x + 6$ is

a) $(4,3)$

b) $(0,1)$

c) $(1,\sqrt{3})$

d) $(4,-3)$

e) $(2,\sqrt{5})$

17. $f'(x) = x\sin x - \cos x$ for $0 < x < 4$. f has a local maximum when x is approximately

a) 0.9

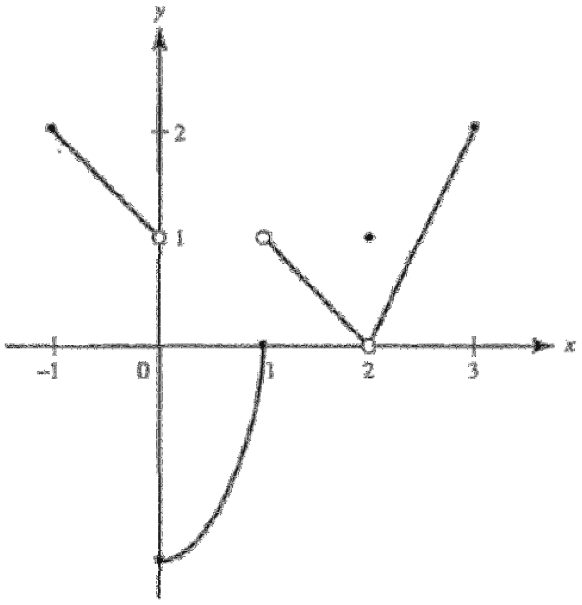
b) 1.2

c) 2.3

d) 3.4

e) 3.7

18. Based on the function *f* shown in the graph below.



The function *f* is defined on [−1,3]

a) if $x \neq 0$.

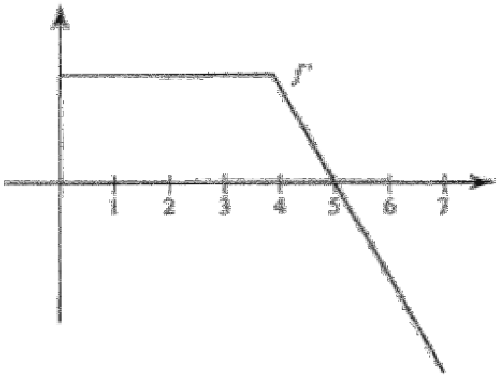
b) if $x \neq 1$.

c) if $x \neq 2$.

d) if $x \neq 3$.

e) at each x in [−1,3].

19. The graph of f' is shown below.



Which statement best describe f at $x = 5$?

a) f has a root.

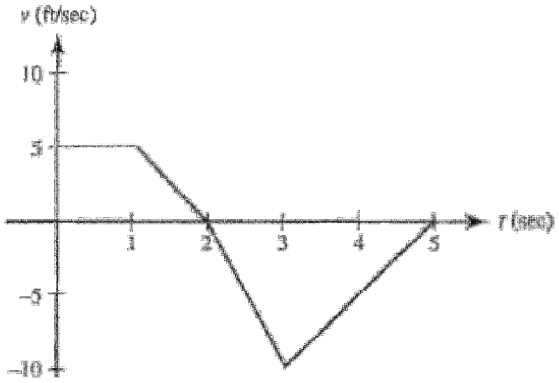
b) f has a maximum.

c) f has a minimum.

d) f has a point of inflection.

e) none of these

20. The graph below shows the velocity of an object moving along a straight line during the time interval $0 \le t \le 5$.



The object is furthermost to the right when $t =$

a) 0

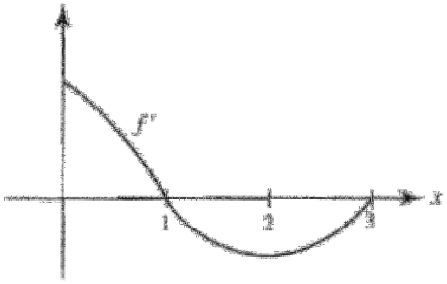
b) 1

c) 2

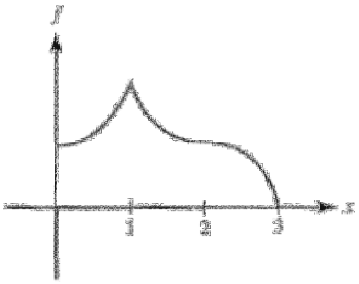
d) 3

e) 5

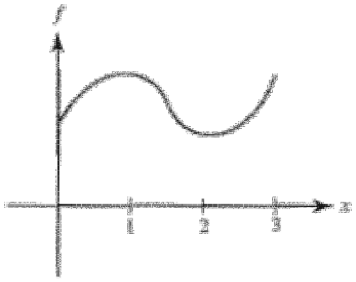
21. Given f' as graphed, which could be the graph of f?



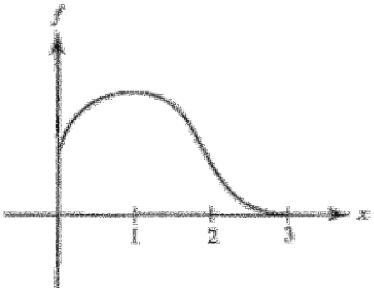
a)



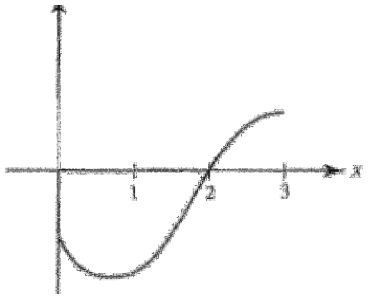
b)



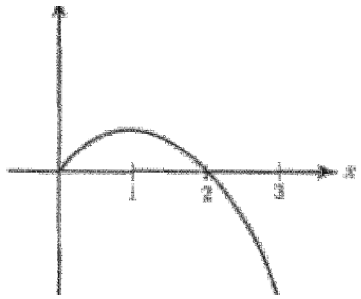
c)



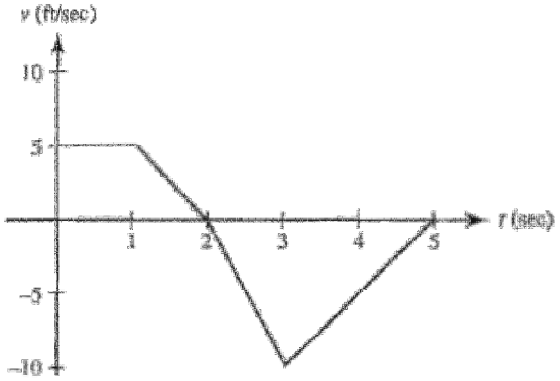
d)



e)



22. The graph below shows the velocity of an object moving along a straight line during the time interval $0 \le t \le 5$.



The speed of the object is increasing during the time interval

a) $(0,1)$

b) $(1,2)$

c) $(0,2)$

d) $(2,3)$

e) $(3,5)$

23. The table below shows the velocity at time t of an object moving along a line. Estimate the acceleration (in ft/sec^2) at $t = 6$ sec.

t (sec)	0	4	8	10
vel.	18	16	10	0

a) -6

b) -1.8

c) -1.5

d) 1.5

e) 6

24. $\lim_{x \to 0} \sqrt{3 + \arctan \frac{1}{x}}$ is

a) $-\infty$

b) $\sqrt{3 - \frac{\pi}{2}}$

c) $\sqrt{3 + \frac{\pi}{2}}$

d) ∞

e) none of these

25. The table below shows some points on a function f that is both continuous and differentiable on the closed interval $[2, 10]$.

x	2	4	6	8	10
$f(x)$	30	25	20	25	30

Which must be true?

a) $f(x) > 0$ for $2 < x < 10$.

b) $f'(6) = 0$.

c) $f'(8) > 0$.

d) The maximum value of f on the interval $[2, 10]$ is 30.

e) For some value of x on the interval $[2, 10]$ $f'(x) = 0$.

Essay

A graphing calculator is REQUIRED for some questions on this part of the exam. (15 minutes)

26. 2007 Form B #3

The wind chill is the temperature in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is 32°F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Fine $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the timer interval $0 \le v \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

A calculator may NOT be used on this part of the exam. (15 minutes)

27. 2006 #6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions: $f(0) = 2$, $f'(0) = -4$, and $f''(0) = 3$.

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

2013-2014 AP Calculus AB Unit 3 Assessment

Answer Section

MULTIPLE CHOICE

1.	ANS: C	DIF:	DOK.2	STA:	C 4.4
2.	ANS: C	DIF:	DOK.4	STA:	C 12.0
3.	ANS: E	DIF:	DOK.1	STA:	C 9.0
4.	ANS: B	DIF:	DOK.3	STA:	C 4.2
5.	ANS: D	DIF:	DOK.4	STA:	C 12.0
6.	ANS: C	DIF:	DOK.3	STA:	C 4.1
7.	ANS: B	DIF:	DOK.1	STA:	C 1.1
8.	ANS: D	DIF:	DOK.4	STA:	C 12.0
9.	ANS: E	DIF:	DOK.2	STA:	C 4.1
10.	ANS: B	DIF:	DOK.2	STA:	C 2.0
11.	ANS: A	DIF:	DOK.3	STA:	C 11.0
12.	ANS: D	DIF:	DOK.2	STA:	C 9.0
13.	ANS: E	DIF:	DOK.4	STA:	C 8.0
14.	ANS: E	DIF:	DOK.3	STA:	C 11.0
15.	ANS: E	DIF:	DOK.2	STA:	C 1.1
16.	ANS: A	DIF:	DOK.2	STA:	C 4.1
17.	ANS: D	DIF:	DOK.2	STA:	C 9.0
18.	ANS: E	DIF:	DOK.2	STA:	C 2.0
19.	ANS: B	DIF:	DOK.2	STA:	C 9.0
20.	ANS: C	DIF:	DOK.2	STA:	C 4.2
21.	ANS: C	DIF:	DOK.4	STA:	C 9.0
22.	ANS: D	DIF:	DOK.2	STA:	C 4.2
23.	ANS: C	DIF:	DOK.1	STA:	C 4.2
24.	ANS: E	DIF:	DOK.2	STA:	C 1.2
25.	ANS: E	DIF:	DOK.3	STA:	C 4.3

ESSAY

26. ANS:

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at 0.286 °F/mph.

2 : { 1 : value; 1 : explanation }

(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\frac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254.

$W'(v) = \frac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

3 : { 1 : average rate of change; 1 : $W'(v)$ = average rate of change; 1 : value of v }

(c) $\left.\frac{dW}{dt}\right|_{t=3} = \left(\frac{dW}{dv} \cdot \frac{dv}{dt}\right)\Big|_{t=3} = W'(35) \cdot 5 = -0.892$ °F/hr

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\left.\frac{dW}{dt}\right|_{t=3} = -0.892$ °F/hr

3 : { 1 : $\frac{dv}{dt} = 5$; 1 : uses $v(3) = 35$, or uses $v(t) = 20 + 5t$; 1 : answer }

Units of °F/mph in (a) and °F/hr in (c)

1 : units in (a) and (c)

DIF: DOK.4 STA: C 4.2 / C 8.0 / C 12.0

27. ANS:

(a) $g'(x) = ae^{ax} + f'(x)$

$g'(0) = a - 4$

$g''(x) = a^2e^{ax} + f''(x)$

$g''(0) = a^2 + 3$

4 : { 1 : $g'(x)$; 1 : $g'(0)$; 1 : $g''(x)$; 1 : $g''(0)$ }

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx)f(x)$

$h'(0) = f'(0)\cos(0) - k\sin(0)f(0) = f'(0) = -4$

$h(0) = \cos(0)f(0) = 2$

The equation of the tangent line is $y = -4x + 2$.

5 : { 2 : $h'(x)$; 3 : { 1 : $h'(0)$; 1 : $h(0)$; 1 : equation of tangent line } }

DIF: DOK.4 STA: C 4.1 / C 9.0