Name: ________________________ **Class:** ___________________ **Date:** __________ **ID: A**

2013-2014 AP Calculus AB Unit 2 Assessment

Multiple Choice
Identify the choice that best completes the statement or answers the question.

A calculator may NOT be used on this part of the exam. (24 minutes)

1. The graph of $f(x) = \dfrac{4}{x^2 - 1}$ has

a) one vertical asymptote at $x = 1$.

b) the y-axis as vertical asymptote.

c) the x-axis as horizontal asymptote and $x = \pm 1$ as vertical asymptotes.

d) two vertical asymptotes at $x = \pm 1$ but no horizontal asymptote.

e) no asymptote.

2. If $f(x) = \ln x^3$, then $f''(3)$ is

a) $-\dfrac{1}{3}$

b) -1

c) -3

d) 1

e) none of these

3. $\lim\limits_{h \to 0} \dfrac{(1+h)^6 - 1}{h}$ is

a) 0

b) 1

c) 6

d) ∞

e) nonexistent

4. If $y = \dfrac{x^2}{\cos x}$, then $\dfrac{dy}{dx} =$

a) $\dfrac{2x}{\sin x}$

b) $\dfrac{-2x}{\sin x}$

c) $\dfrac{2x\cos x - x^2 \sin x}{\cos^2 x}$

d) $\dfrac{2x\cos x + x^2 \sin x}{\cos^2 x}$

e) $\dfrac{2x\cos x + x^2 \sin x}{\sin^2 x}$

5. If $y = x^5 \tan x$, then $\dfrac{dy}{dx} =$

a) $5x^4 \tan x$

b) $x^5 \sec^2 x$

c) $5x^4 \sec^2 x$

d) $5x^4 + \sec^2 x$

e) $5x^4 \tan x + x^5 \sec^2 x$

6. $\lim\limits_{x \to \infty} \dfrac{3x^2 + 27}{x^3 - 27}$ is

a) 3

b) ∞

c) 1

d) -1

e) 0

7. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0,5)$, $\dfrac{d^2y}{dx^2}$ is

a) 0

b) $\dfrac{1}{5}$

c) -5

d) $-\dfrac{1}{5}$

e) nonexistent

8. If $\sin xy = x$, then $\dfrac{dy}{dx} =$

a) $\sec xy$

b) $\dfrac{\sec xy}{x}$

c) $\dfrac{\sec xy - y}{x}$

d) $-\dfrac{1 + \sec xy}{x}$

e) $\sec xy - 1$

9. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

a) -32

b) -16

c) -4

d) -2

e) $-\dfrac{1}{2}$

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10. If $y = \tan^{-1} \frac{x}{2}$, then $\frac{dy}{dx} =$

a) $\frac{4}{4+x^2}$

b) $\frac{1}{2\sqrt{4-x^2}}$

c) $\frac{2}{\sqrt{4-x^2}}$

d) $\frac{1}{2+x^2}$

e) $\frac{2}{x^2+4}$

11. $\lim_{x \to 0} \frac{\sin x}{x^2 + 3x}$ is

a) 1

b) $\frac{1}{3}$

c) 3

d) ∞

e) $\frac{1}{4}$

12. Suppose $f(1) = 2$, $f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

a) equals $-\frac{1}{3}$.

b) equals $-\frac{1}{4}$.

c) equals $\frac{1}{3}$.

d) equals $\frac{1}{4}$.

e) cannot be determined.

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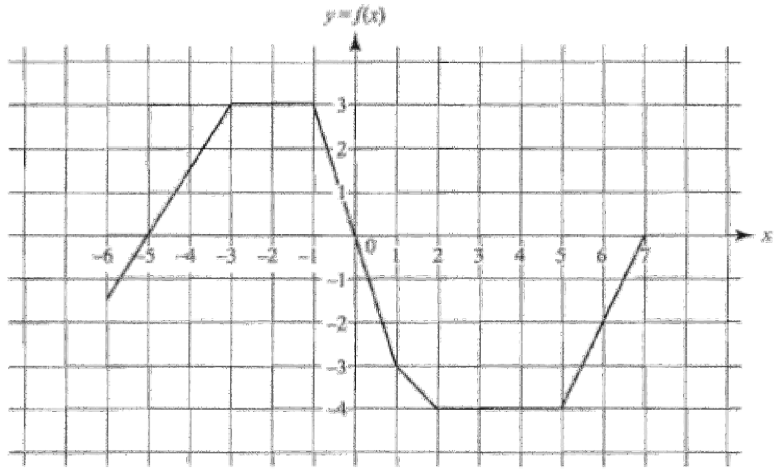
A graphing calculator is REQUIRED for some questions on this part of the exam. (24 minutes)

13. Use the graph of $f(x)$ sketched below on the interval $-6 \le x \le 7$.



Which of the following statements about the graph of $f'(x)$ is false?

a) It consists of six horizontal segments.

b) It has four jump discontinuities.

c) $f'(x)$ is discontinuous at each x in the set $\{-3, -1, 1, 2, 5\}$.

d) $f'(x)$ ranges from -3 to 2.

e) On the interval $-1 < x < 1, f'(x) = -3$.

14. At how many points on the interval $[-5, 5]$ is a tangent to $y = x + \cos x$ parallel to the secant line?

a) none

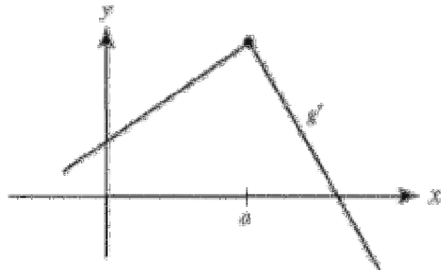
b) 1

c) 2

d) 3

e) more than 3

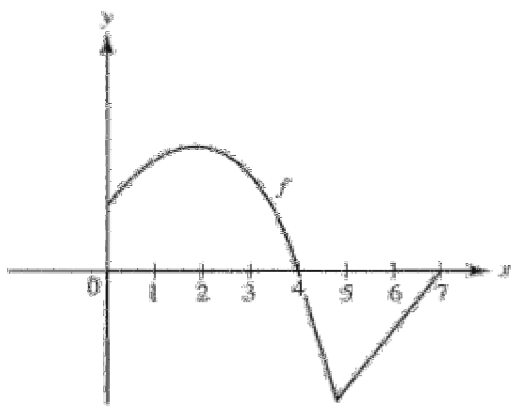
15. The graph of g' is shown below. Which of the following statements is (are) true of g at $x = a$?



I. g is continuous.
II. g is differentiable.
III. g is increasing.

a) I only
b) III only
c) I and III only
d) II and III only
e) I, II, and III

16. The function f whose graph is shown has $f' = 0$ at $x =$



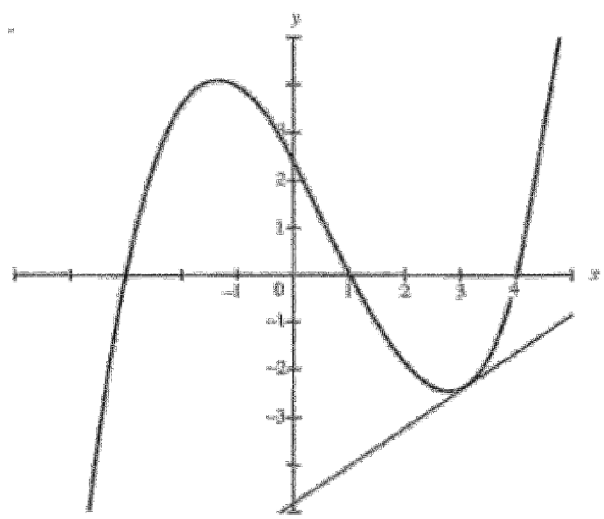
a) 2 only
b) 2 and 5
c) 4 and 7
d) 2, 4, and 7
e) 2, 4, 5, and 7

17. A differentiable function f has the values shown below. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

a) 8
b) 12
c) 18
d) 40
e) 80

18. Using the graph below, the rate of change of $f(x)$ is least at $x \approx$



a) −3
b) −1.3
c) 0
d) 0.7
e) 2.7

19. Suppose $\lim_{x \to -3^-} f(x) = -1$, $\lim_{x \to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statements is (are) true?

    I. $\lim_{x \to -3} f(x) = -1$.

    II. f is continuous everywhere except at $x = -3$.

    III. f has a removable discontinuity at $x = -3$.

    a) None of them
    b) I only
    c) III only
    d) I and III only
    e) All of them

20. Let $f(x) = 3^x - x^3$. The tangent to the curve is parallel to the secant through $(0,1)$ and $(3,0)$ for $x =$

    a) 0.984 only
    b) 1.244 only
    c) 2.727 only
    d) 0.984 and 2.804 only
    e) 1.244 and 2.727 only

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Essay

A graphing calculator is REQUIRED for some questions on this part of the exam. (15 minutes)

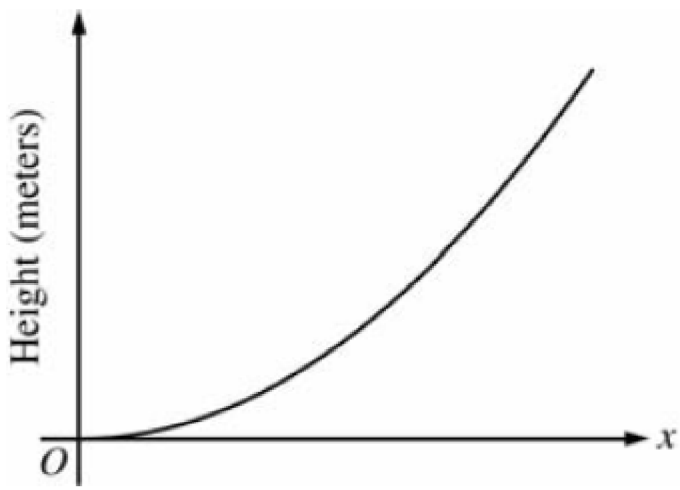
21. 2006 Form B #3

The figure below is the graph of a function of x, which models the height of a skateboard ramp. The function meets the following requirements.

(i) At $x = 0$, the values of the function is 0, and the slope of the graph of the function is 0.

(ii) At $x = 4$, the values of the function is 1, and the slope of the graph of the function is 1.

(iii) Between $x = 0$ and $x = 4$, the function is increasing.



(a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.

(b) Let $g(x) = cx^3 - \frac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirements (ii) above. Show the work that leads to your answer.

(c) Using the function g and your value c from part (b), show that g does not meet requirement (iii) above.

(d) Let $h(x) = \frac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirements (ii) above. Show that h also meets requirements (i) and (iii) above.

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A calculator may NOT be used on this part of the exam. (15 minutes)

22. 2008 Form B #6

Consider the closed curve in the *xy*-plane given by $x^2 + 2x + y^4 + 4y = 5$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2(y^3+1)}$

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the *x*-axis? Explain your reasoning.

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Answer Section

MULTIPLE CHOICE

1.	ANS:	C	DIF:	DOK.3	STA:	C 2.0
2.	ANS:	A	DIF:	DOK.2	STA:	C 7.0
3.	ANS:	C	DIF:	DOK.1	STA:	C 4.1
4.	ANS:	D	DIF:	DOK.1	STA:	C 4.4
5.	ANS:	E	DIF:	DOK.1	STA:	C 4.4
6.	ANS:	E	DIF:	DOK.2	STA:	C 1.1
7.	ANS:	D	DIF:	DOK.2	STA:	C 6.0
8.	ANS:	C	DIF:	DOK.2	STA:	C 5.0
9.	ANS:	E	DIF:	DOK.2	STA:	C 7.0
10.	ANS:	E	DIF:	DOK.1	STA:	C 4.4
11.	ANS:	B	DIF:	DOK.2	STA:	C 1.3
12.	ANS:	C	DIF:	DOK.2	STA:	C 4.4
13.	ANS:	B	DIF:	DOK.3	STA:	C 4.3
14.	ANS:	D	DIF:	DOK.2	STA:	C 4.1
15.	ANS:	E	DIF:	DOK.3	STA:	C 4.3
16.	ANS:	A	DIF:	DOK.2	STA:	C 4.3
17.	ANS:	D	DIF:	DOK.2	STA:	C 4.1
18.	ANS:	D	DIF:	DOK.2	STA:	C 4.1
19.	ANS:	D	DIF:	DOK.3	STA:	C 2.0
20.	ANS:	E	DIF:	DOK.2	STA:	C 4.1

ESSAY

21. ANS:

(a) $f(4) = 1$ implies that $a = \frac{1}{16}$ and $f'(4) = 2a(4) = 1$ implies that $a = \frac{1}{8}$. Thus, f cannot satisfy (ii).

$2 : \begin{cases} 1 : a = \frac{1}{16} \text{ or } a = \frac{1}{8} \\ 1 : \text{shows } a \text{ does not work} \end{cases}$

(b) $g(4) = 64c - 1 = 1$ implies that $c = \frac{1}{32}$.

When $c = \frac{1}{32}$, $g'(4) = 3c(4)^2 - \frac{2(4)}{16} = 3\left(\frac{1}{32}\right)(16) - \frac{1}{2} = 1$

$1 :$ value of c

(c) $g'(x) = \frac{3}{32}x^2 - \frac{x}{8} = \frac{1}{32}x(3x - 4)$

$g'(x) < 0$ for $0 < x < \frac{4}{3}$, so g does not satisfy (iii).

$2 : \begin{cases} 1 : g'(x) \\ 1 : \text{explanation} \end{cases}$

(d) $h(4) = \frac{4^n}{k} = 1$ implies that $4^n = k$.

$h'(4) = \frac{n4^{n-1}}{k} = \frac{n4^{n-1}}{4^n} = \frac{n}{4} = 1$ gives $n = 4$ and $k = 4^4 = 256$.

$h(x) = \frac{x^4}{256} \Rightarrow h(0) = 0.$

$h'(x) = \frac{4x^3}{256} \Rightarrow h'(0) = 0$ and $h'(x) > 0$ for $0 < x < 4$.

$4 : \begin{cases} 1 : \frac{4^n}{k} = 1 \\ 1 : \frac{n4^{n-1}}{k} = 1 \\ 1 : \text{values for } k \text{ and } n \\ 1 : \text{verifications} \end{cases}$

DIF: DOK.4 STA: C 4.1 / C 4.4

22. ANS:

(a) $2x + 2 + 4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$\left(4y^3 + 4\right)\frac{dy}{dx} = -2x - 2$

$\frac{dy}{dx} = \frac{-2(x+1)}{4\left(y^3+1\right)} = \frac{-(x+1)}{2\left(y^3+1\right)}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(-2,\,1)} = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4}$

Tangent line: $y = 1 + \frac{1}{4}(x+2)$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

$3 : \begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1 : \text{answer} \end{cases}$

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

The curve crosses the x-axis where $y = 0$.

$(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$

No, the curve cannot have a horizontal tangent where it crosses the x-axis.

$2 : \begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$

DIF: DOK.4 STA: C 4.1 / C4.3 / C 6.0