

Derivatives and Integrals of Inverse Trig Functions

Differentiate each function with respect to x .

1) $f(x) = \tan^{-1}(5x^4 + 3)^3$

2) $f(x) = \csc^{-1} 2x^3$

3) $y = (\cot^{-1} 5x^5)^4$

4) $y = \sin^{-1} 3x^3$

5) $y = \cos^{-1} 2x^2$

6) $f(x) = \sec^{-1}(x^2 + 1)^5$

Evaluate each indefinite integral.

$$7) \int \frac{1}{4+x^2} dx$$

$$8) \int \frac{1}{x\sqrt{x^2-9}} dx$$

$$9) \int \frac{1}{x\sqrt{x^2-25}} dx$$

$$10) \int \frac{1}{\sqrt{9-x^2}} dx$$

$$11) \int \frac{2e^{2x}}{\sqrt{25-e^{4x}}} dx$$

$$12) \int \frac{9x^2}{25+9x^6} dx$$

$$13) \int \frac{3e^{3x}}{e^{3x}\sqrt{e^{6x}-1}} dx$$

$$14) \int \frac{12x^2}{4x^3\sqrt{16x^6-9}} dx$$

Derivatives and Integrals of Inverse Trig Functions Date _____ Period _____

Differentiate each function with respect to x .

1) $f(x) = \tan^{-1}(5x^4 + 3)^3$

$$\begin{aligned} f'(x) &= \frac{1}{((5x^4 + 3)^3)^2 + 1} \cdot 3(5x^4 + 3)^2 \cdot 20x^3 \\ &= \frac{60x^3(5x^4 + 3)^2}{(5x^4 + 3)^6 + 1} \end{aligned}$$

2) $f(x) = \csc^{-1} 2x^3$

$$\begin{aligned} f'(x) &= -\frac{1}{|2x^3| \sqrt{(2x^3)^2 - 1}} \cdot 6x^2 \\ &= -\frac{6x^2}{|2x^3| \sqrt{4x^6 - 1}} \end{aligned}$$

3) $y = (\cot^{-1} 5x^5)^4$

$$\begin{aligned} \frac{dy}{dx} &= 4(\cot^{-1} 5x^5)^3 \cdot -\frac{1}{(5x^5)^2 + 1} \cdot 25x^4 \\ &= -\frac{100x^4(\cot^{-1} 5x^5)^3}{25x^{10} + 1} \end{aligned}$$

4) $y = \sin^{-1} 3x^3$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{1 - (3x^3)^2}} \cdot 9x^2 \\ &= \frac{9x^2}{\sqrt{1 - 9x^6}} \end{aligned}$$

5) $y = \cos^{-1} 2x^2$

$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{\sqrt{1 - (2x^2)^2}} \cdot 4x \\ &= -\frac{4x}{\sqrt{1 - 4x^4}} \end{aligned}$$

6) $f(x) = \sec^{-1}(x^2 + 1)^5$

$$\begin{aligned} f'(x) &= \frac{1}{|(x^2 + 1)^5| \sqrt{((x^2 + 1)^5)^2 - 1}} \cdot 5(x^2 + 1)^4 \cdot 2x \\ &= \frac{10x}{\sqrt{(x^2 + 1)^{10} - 1}(x^2 + 1)} \end{aligned}$$

Evaluate each indefinite integral.

$$7) \int \frac{1}{4+x^2} dx$$
$$\frac{1}{2} \cdot \tan^{-1} \frac{x}{2} + C$$

$$8) \int \frac{1}{x\sqrt{x^2-9}} dx$$
$$\frac{1}{3} \cdot \sec^{-1} \frac{x}{3} + C$$

$$9) \int \frac{1}{x\sqrt{x^2-25}} dx$$
$$\frac{1}{5} \cdot \sec^{-1} \frac{x}{5} + C$$

$$10) \int \frac{1}{\sqrt{9-x^2}} dx$$
$$\sin^{-1} \frac{x}{3} + C$$

$$11) \int \frac{2e^{2x}}{\sqrt{25-e^{4x}}} dx$$
$$\sin^{-1} \frac{e^{2x}}{5} + C$$

$$12) \int \frac{9x^2}{25+9x^6} dx$$
$$\frac{1}{5} \cdot \tan^{-1} \frac{3x^3}{5} + C$$

$$13) \int \frac{3e^{3x}}{e^{3x}\sqrt{e^{6x}-1}} dx$$
$$\sec^{-1} e^{3x} + C$$

$$14) \int \frac{12x^2}{4x^3\sqrt{16x^6-9}} dx$$
$$\frac{1}{3} \cdot \sec^{-1} \frac{4x^3}{3} + C$$