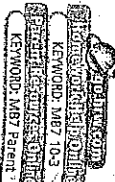California Standards
16.0, 24.0

GUIDED PRACTICE

1. **Vocabulary** How can you tell the difference between the *major axis* of an ellipse and the *minor axis* of an ellipse?

Find the constant sum of an ellipse with the given foci and point on the ellipse.

2. $F_1(-5, 0), F_2(5, 0), P(0, -12)$

3. $F_1(0, -12), F_2(0, 12), P(9, 0)$

Write an equation in standard form for each ellipse with center $(0, 0)$.

4. vertex $(-9, 0)$, co-vertex $(0, 7)$

5. vertex $(0, 25)$, focus $(0, -20)$

6. co-vertex $(10, 0)$, focus $(0, 24)$

7. vertex $(-7, 0)$, focus $(\sqrt{13}, 0)$

Graph each ellipse.

8. $\dfrac{x^2}{36} + \dfrac{y^2}{81} = 1$

9. $\dfrac{x^2}{121} + \dfrac{y^2}{49} = 1$

10. $\dfrac{(x-5)^2}{16} + \dfrac{(y+2)^2}{36} = 1$

11. $\dfrac{(x+1)^2}{64} + \dfrac{(y-6)^2}{9} = 1$

12. **Engineering** Engineers are building semi-elliptical bridges across two rivers. The larger river is 4 times as wide as the smaller river and must accommodate boats that are 3 times as tall. The equation for the bridge over the smaller river is $\dfrac{x^2}{25} + \dfrac{y^2}{144} = 1$, measured in feet.

   a. Find the dimensions of the larger bridge.

   b. Write an equation for the design of the larger bridge.

PRACTICE AND PROBLEM SOLVING

Find the constant sum of an ellipse with the given foci and point on the ellipse.

13. $F_1(-20, 0), F_2(20, 0), P(-21, 0)$

14. $F_1(0, -8), F_2(0, 8), P(9, 13.6)$

Write an equation in standard form for each ellipse with center $(0, 0)$.

15. vertex $(5, 0)$, co-vertex $(0, -2)$

16. co-vertex $(0, -8)$, focus $(6, 0)$

17. co-vertex $(4, 0)$, focus $(0, -3)$

18. vertex $(0, -9)$, focus $(0, 3\sqrt{5})$

Graph each ellipse.

19. $\dfrac{(x+2)^2}{169} + \dfrac{(y-7)^2}{25} = 1$

20. $\dfrac{(x-6)^2}{36} + \dfrac{(y-4)^2}{100} = 1$

21. $\dfrac{x^2}{256} + \dfrac{y^2}{196} = 1$

22. $\dfrac{x^2}{225} + \dfrac{y^2}{289} = 1$

23. **National Parks** South of the White House in Washington, D.C., is the President's Park South, or the Ellipse, which hosts events such as the White House Garden Tours. The Ellipse is 880 ft from north to south and 1057 ft from east to west. Write an equation for the Ellipse, centered at the origin.

Write an equation in standard form for each ellipse.

24. tangent to the x-axis at $(9, 0)$ and tangent to the y-axis at $(0, -6)$

25. center $(-4, 7)$, vertex $(-4, -3)$, focus $(-4, 0)$

26. **Estimation** An ellipse has a vertex at the point $(2.4, -6.1)$, focus $(0.35, -6.1)$, and center $(-4.5, -6.1)$. Estimate the coordinates of the co-vertices.

Write an equation for each graph, and give the domain and range. (*Hint:* The domain and range depend on the center and the lengths of the major and minor axes.)

27.

28.

29.
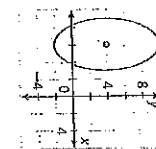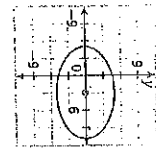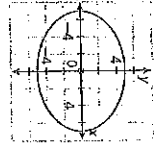
31. **History** The Roman Colosseum is shaped like a large ellipse, with an external width of 188 m and a length of 156 m. Write an equation that can be used to model the shape of the Colosseum.
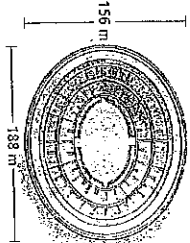
30. **Architecture** As a result of their unique elliptical shapes, whispering galleries enable even the smallest sound generated at one focus to be carried across the room to the other focus. The whispering gallery at the Chicago Museum of Science and Industry is 47 ft 4 in. long and 13 ft 6 in. wide.

   a. Supposing that the center of the floor of the whispering gallery is located at the origin, write an equation for the gallery floor.

   b. Find the coordinates of the foci. How far apart are they?

Find the center, vertices, co-vertices, foci, domain, and range of each ellipse.

32. $\dfrac{(x-1)^2}{225} + \dfrac{(y+5)^2}{324} = 1$

33. $9(x+9)^2 + 81(y+4)^2 = 729$

34. **Critical Thinking** An ellipse is defined by the distance $PF_1 + PF_2 = d$. Could the distance between the foci be less than $PF_1 + PF_2$? Explain.

35. **Geometry** The area of an ellipse in standard form is given by $A = \pi ab$.

   a. **Critical Thinking** How is the formula for the area of an ellipse related to the formula for the area of a circle?

   b. Find the area of $\dfrac{(x+2)^2}{169} + \dfrac{(y-7)^2}{25} = 1$.

36. This problem will prepare you for the Concept Connection on page 758.

   The figure shows the elliptical orbit of Mars, where each unit of the coordinate plane represents 1 million kilometers. As shown, the planet's maximum distance from the Sun is 249 million kilometers and its minimum distance from the Sun is 207 million kilometers.

   a. The Sun is at one focus of the ellipse. What are the coordinates of the Sun?

   b. What is the length of the minor axis of the ellipse?

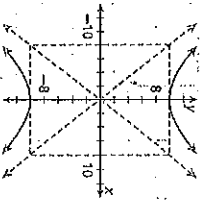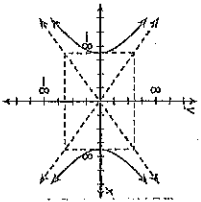   c. Write an equation that models the orbit of Mars.

Exercises
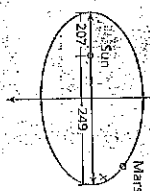
GUIDED PRACTICE

1. **Vocabulary** The vertices of a hyperbola lie on the _ ? _ (*transverse axis* or *conjugate axis*).

SEE EXAMPLE 1 p. 744

Find the constant difference for a hyperbola with the given foci and point on the hyperbola.

2. $F_1(-13, 0), F_2(13, 0), P(5, 0)$

3. $F_1(0, -17), F_2(0, 17), P(0, -15)$

SEE EXAMPLE 2 p. 745

Write an equation in standard form for each hyperbola.

4. center $(0, 0)$, vertex $(0, 5)$, and focus $(0, 13)$

5. center $(0, 0)$, vertex $(9, 0)$, and co-vertex $(0, 7)$

SEE EXAMPLE 3 p. 746

6.

7.

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

8. $\dfrac{x^2}{49} - \dfrac{y^2}{36} = 1$

9. $\dfrac{x^2}{25} - \dfrac{y^2}{64} = 1$

10. $\dfrac{y^2}{25} - \dfrac{x^2}{36} = 1$

11. $\dfrac{y^2}{100} - \dfrac{x^2}{81} = 1$

12. $\dfrac{(x - 4)^2}{25} - \dfrac{(y - 3)^2}{9} = 1$

13. $\dfrac{(x - 4)^2}{16} - \dfrac{(y + 6)^2}{49} = 1$

14. $\dfrac{(y + 8)^2}{36} - \dfrac{(x + 3)^2}{25} = 1$

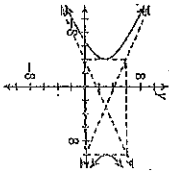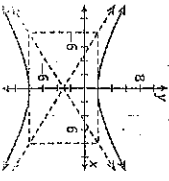15. $\dfrac{(y + 7)^2}{4} - \dfrac{x^2}{25} = 1$

PRACTICE AND PROBLEM SOLVING

Find the constant difference for a hyperbola with the given foci and point on the hyperbola.

16. $F_1(0, -10), F_2(0, 10), P(0, 6)$

17. $F_1(-29, 0), F_2(29, 0), P(21, 0)$

Write an equation in standard form for each hyperbola.

18. center $(0, 0)$, vertex $(15, 0)$, co-vertex $(0, -13)$

19. center $(0, 0)$, vertex $(-8, 0)$, focus $(17, 0)$

20.

21.

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

22. $\dfrac{x^2}{64} - \dfrac{y^2}{36} = 1$

23. $\dfrac{y^2}{25} - \dfrac{x^2}{81} = 1$

24. $\dfrac{y^2}{81} - \dfrac{x^2}{16} = 1$

25. $\dfrac{x^2}{4} - \dfrac{y^2}{121} = 1$

26. $\dfrac{(y - 1)^2}{64} - \dfrac{(x + 2)^2}{36} = 1$

27. $\dfrac{(x + 5)^2}{25} - \dfrac{(y - 3)^2}{16} = 1$

28. $\dfrac{(y - 8)^2}{25} - \dfrac{(x + 6)^2}{36} = 1$

29. $\dfrac{(x - 6)^2}{9} - \dfrac{(y - 2)^2}{16} = 1$

30. **Architecture** If the x-axis is placed at a height of 100 meters, the outer edge of a cooling tower can be modeled by the hyperbola $\dfrac{x^2}{900} - \dfrac{y^2}{1600} = 1$, measured in meters. If the tower is 150 meters tall, find the width of the cooling tower at the top.
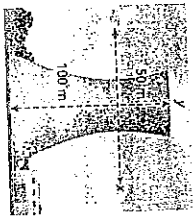
31. **Critical Thinking** What happens to the graph of $\dfrac{x^2}{a^2} - \dfrac{y^2}{16} = 1$ as the values of a increase? What happens to the graph of $\dfrac{x^2}{16} - \dfrac{y^2}{b^2} = 1$ as the values of b increase?

The saying "lightning never strikes twice in the same place" is often disproven. The Empire State Building is struck by lightning about 100 times each year and serves as a lightning rod for the surrounding area.

32. **Physics** Two people standing 10,000 feet apart see lightning strike. One person hears the thunder 5 seconds after the other person. Because sound travels at 1100 feet per second, one person is 5500 feet farther from the lightning strike than the other. The possible locations of the strike then form a hyperbola with the two people at the foci. Place the origin midway between the two people, and write an equation that could be used to represent the possible locations of the lightning strike.

33. **Biology** Two underwater listening devices 12,000 feet apart detect a whale call. One device detects the call 2 seconds before the other. The possible locations of the whale form a hyperbola with the two devices at the foci.

a. If the speed of sound in water is 5000 feet per second, write an equation for the possible locations of the whale. [*Hint:* Place the origin midway between the devices.]

b. **What If...?** Could the location of the whale be more precisely located if there were a third listening device? Explain.

34. **Critical Thinking** How could you identify the domain and range of a hyperbola? Explain.

35. **Critical Thinking** Consider a hyperbola with equation $\dfrac{(y - b)^2}{a^2} - \dfrac{(x - b)^2}{b^2} = 1$. Which parameter—$a$, b, or c—has the greatest value? Which has the least value? Explain.

36. **Write About It** Suppose you have two hyperbolas that are the same except that the transverse axis and conjugate axis are switched. How does switching the axes affect the equations of the asymptotes for the two hyperbolas? Why?

37. This problem will prepare you for the Concept Connection on page 758.

A comet's path as it approaches the Sun is modeled by one branch of the hyperbola $\dfrac{y^2}{500} - \dfrac{x^2}{44,999} = 1$, where the Sun is at the corresponding focus. Each unit of the coordinate plane represents 1 million miles.

a. Find the coordinates of the Sun, assuming that it is at the focus with nonnegative coordinates.

b. How close does the comet come to the Sun?

c. When the comet is far from the Sun, the comet's path can be modeled by the hyperbola's asymptotes. Write the equations of the asymptotes.