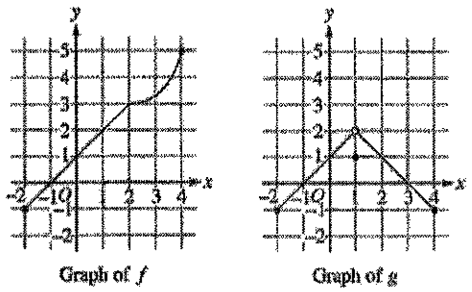
Collegeboard Exam Description Practice Questions



1. The graphs of the functions f and g are shown above. The value of $\lim_{x\to 1} f(g(x))$ is

(A) 1

(B) 2

(C) 3

(D) nonexistent

$\lim_{x\to 1} g(x) = 2$

$\lim_{x\to 1} f(2) = 3$

2. $\lim_{x\to 0} \frac{7x - \sin x}{x^2 + \sin(3x)} = \frac{0}{0}$

(A) 6

(B) 2

(C) 1

(D) 0

$\lim_{x\to 0} \frac{7 - \cos x}{2x + \cos(3x)\cdot 3}$

$\frac{7 - \cos(0)}{\cos(0)\cdot 3} = \frac{7-1}{3} = 2$

3. If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

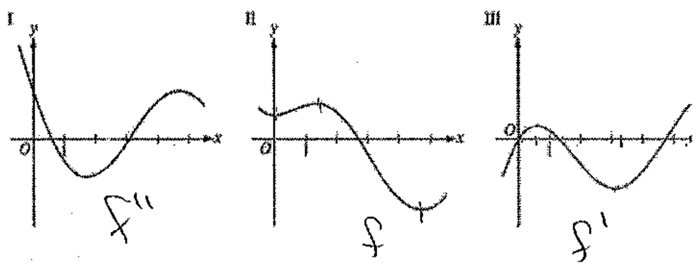
(A) $\frac{\sin(\ln(2x))}{2x}$

(B) $\frac{\cos(\ln(2x))}{x}$

(C) $\frac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\frac{1}{2x}\right)$

$f'(x) = \cos(\ln(2x))\frac{1}{2x}\cdot 2$



4. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	II	I	III
(C)	II	III	I
(D)	III	I	II

tan line approx

$g(\frac{1}{2}) = 4(\frac{1}{2}) + 1 = 3$

pt of tan is same as $g(\frac{1}{2})$

5. The local linear approximation to the function g at $x = \frac{1}{2}$ is $y = 4x + 1$. What is the value of $g\left(\frac{1}{2}\right) + g'\left(\frac{1}{2}\right)$?

(A) 4

(B) 5

(C) 6

(D) 7

$g'(\frac{1}{2}) = 4$

$3 + 4 = 7$

6. For time $t \geq 0$, the velocity of a particle moving along the x-axis is given by $v(t) = (t-5)(t-2)^2$. At what values of t is the acceleration of the particle equal to 0?

(A) 2 only

(B) 4 only

(C) 2 and 4

(D) 2 and 5

$a(t) = (t-5)(2(t-2)) + (t-2)^2$

$= (t-2)(2(t-5) + (t-2))$

$= (t-2)(2t - 10 + t - 2)$

$0 = (t-2)(3t-12) \quad t=2, t=4$

7. The cost, in dollars, to shred the confidential documents of a company is modeled by C, a differentiable function of the weight of documents in pounds. Of the following, which is the best interpretation of $C'(500) = 80$?

$\to$ rate

(A) The cost to shred 500 pounds of documents is \$80.

(B) The average cost to shred documents is $\frac{80}{500}$ dollar per pound.

(C) Increasing the weight of documents by 500 pounds will increase the cost to shred the documents by approximately \$80.

(D) The cost to shred documents is increasing at a rate of \$80 per pound when the weight of the documents is 500 pounds.

8. Which of the following integral expressions is equal to $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\sqrt{1 + \frac{3k}{n}} \cdot \frac{1}{n}\right)$?

(A) $\int_0^1 \sqrt{1+3x}\, dx$

(B) $\int_0^3 \sqrt{1+x}\, dx$

(C) $\int_1^4 \sqrt{x}\, dx$

(D) $\frac{1}{3}\int_0^3 \sqrt{x}\, dx$

$\Delta x = \frac{b-a}{n}$ $\quad b - a = 1$

right riemann sum, adding $\frac{1}{n}$ each time $(k \cdot \frac{1}{n})$ as k inc.

9. $f(x) = \begin{cases} x & \text{for } x < 2 \\ 3 & \text{for } x \geq 2 \end{cases}$

If f is the function defined above, then $\int_{-1}^{4} f(x)\, dx$ is

(A) $\frac{9}{2}$

(B) $\frac{15}{2}$

(C) $\frac{17}{2}$

(D) undefined

$\int_{-1}^{2} x\, dx + \int_{2}^{4} 3\, dx$

$\left[\frac{x^2}{2}\right]_{-1}^{2} + [3x]_2^4$

$\left(\frac{4}{2} - \frac{1}{2}\right) + (12 - 6)$

$\frac{3}{2} + 6 = \frac{15}{2}$

10. $\int e^x \cos(e^x + 1)\, dx =$

(A) $\sin(e^x + 1) + C$

(B) $e^x \sin(e^x + 1) + C$

(C) $e^x \sin(e^x + x) + C$

(D) $\frac{1}{2}\cos^2(e^x + 1) + C$

$u = e^x + 1$

$du = e^x dx$

$\int e^x \cos(u) \frac{du}{e^x}$

$\sin(e^x + 1) + C$

11. At time t, a population of bacteria grows at the rate of $5e^{0.2t} + 4t$ grams per day, where t is measured in days. By how many grams has the population grown from time $t = 0$ days to $t = 10$ days?

(A) $5e^2 + 40$

(B) $5e^2 + 195$

(C) $25e^2 + 175$

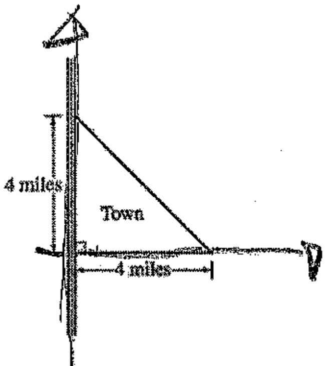
(D) $25e^2 + 375$

$\int_0^{10} 5e^{.2t} + 4t\, dt$

$\left[\frac{5e^{.2t}}{.2} + 2t^2\right]_0^{10}$

$.2 = \frac{1}{5}$

$\left[25e^{.2(10)} + 2(100)\right] - (25)$



12. The right triangle shown in the figure above represents the boundary of a town that is border by a highway. The population density of the town at a distance of x miles from the highway is modeled by $D(x)=\sqrt{x+1}$, where $D(x)$ is measured in thousands of people per square mile According to the model, which of the following expressions gives the total population, in thousands, of the town?

(A) $\int_0^4 \sqrt{x+1}\,dx$

(B) $\int_0^4 8\sqrt{x+1}\,dx$

(C) $\int_0^4 x\sqrt{x+1}\,dx$

(D) $\int_0^4 (4-x)\sqrt{x+1}\,dx$

$\int_0^4 D(x)(4-x)\,dx$

* treat like a cross section question where the "height" is the density

13. Which of the following is the solution to the differential equation $\frac{dy}{dx}=y\sec^2 x$ with initial condition $y\left(\frac{\pi}{4}\right)=-1$?

(A) $y=-e^{\tan x}$

(B) $y=-e^{(-1+\tan x)}$

(C) $y=-e^{(\sec^3 x-2\sqrt{2})/3}$

(D) $y=-\sqrt{2\tan x-1}$

$\int\frac{dy}{y}=\int\sec^2 x\,dx$

$\ln|y|=\tan x+C$

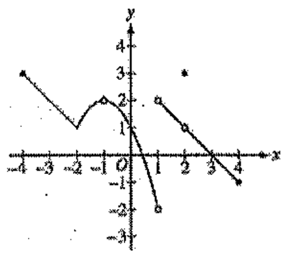
$\ln(1)=\tan\frac{\pi}{4}+C$

$C=-1$

$e^{\ln|y|}=e^{\tan x-1}$

$|y|=e^{\tan x-1}$

$y=-e^{\tan x-1}$



Graph of f

14. The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

(A) one

(B) two

(C) three

(D) four

15.

x	0	1	2
$f(x)$	5	2	−7
$f'(x)$	−2	−5	−14

The table above gives selected values of a differentiable and decreasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(2)$?

(A) $-\frac{1}{5}$

(B) $-\frac{1}{14}$

(C) $\frac{1}{5}$

(D) 5

f: $(1, 2)$, $m=-5$

g: $(2, 1)$, $m=-\frac{1}{5}$

16. The derivative of the function f is given by $f'(x)=-\frac{x}{3}+\cos(x^2)$. At what values of x does f have a relative minimum on the interval $0<x<3$?

(A) 1.094 and 2.608

(B) 1.798

(C) 2.372

(D) 2.493

f' changes from − to +

$t=2.372$

Calculator Questions

17. The second derivative of a function g is given by $g''(x)=2^{-x^2}+\cos x+x$. For $-5<x<5$, on what open intervals is the graph of g concave up?

(A) $-5<x<-1.016$ only

(B) $-1.016<x<5$ only

(C) $0.463<x<2.100$ only

(D) $-5<x<0.463$ and $2.100<x<5$

g'' pos

18. The temperature, in degrees Fahrenheit (°F), of water in a pond is modeled by the function H given by $H(t)=55-9\cos\left(\frac{2\pi}{365}(t+10)\right)$, where t is the number of days since January 1 $(t=0)$. What is the instantaneous rate of change of the temperature of the water at time $t=90$ days?

(A) 0.114°F/day

(B) 0.153°F/day

(C) 50.252°F/day

(D) 56.350°F/day

$H'(90)=.153$

19.

x	0	2	4	8
$f(x)$	3	4	9	13
$f'(x)$	0	1	1	2

The table above gives values of a differentiable function f and its derivative at selected values of x. If h is the function given by $h(x)=f(2x)$, which of the following statements must be true?

$h'(x)=f'(2x)\cdot 2$

(I) h is increasing on $2<x<4$. doesn't have to be true

(II) There exists c, where $0<c<4$, such that $h(c)=12$. ✓ IVT

(III) There exists c, where $0<c<2$, such that $h'(c)=3$. ✓ MVT

(A) II only

(B) I and III only

(C) II and III only

(D) I, II, and III

$h(4)=f(8)=13$ $h(c)=12$

$h(0)=f(0)=3$ IVT

$\frac{h(2)-h(0)}{2-0}=\frac{f(4)-f(0)}{2-0}=\frac{9-3}{2}=\frac{6}{2}=3$

20. Let h be the function defined by $h(x)=\frac{1}{\sqrt{x^5+1}}$. If g is an antiderivative of h and $g(2)=3$, what is the value of $g(4)$?

(A) −0.020

(B) 0.152

(C) 3.031

(D) 3.152

$\int h(t)\,dt=g$

$g(4)=g(2)+\int_2^4\frac{1}{\sqrt{x^5+1}}\,dx$

$=3+.1515$

$=3.152$

Additional Questions for Revised Exam

Noncalculator Questions

1. $\lim_{x\to\pi} \dfrac{\cos x + \sin(2x) + 1}{x^2 - \pi^2}$ is $= \frac{0}{0}$

(A) $\dfrac{1}{2\pi}$ $\lim_{x\to\pi} \dfrac{-\sin x + 2\cos 2x}{2x}$ $\quad \dfrac{-\sin(\pi) + 2\cos 2\pi}{2\pi}$ $\quad \cancel{\dfrac{2}{2\pi}}$

(B) $\dfrac{1}{\pi}$

(C) 1

(D) nonexistent

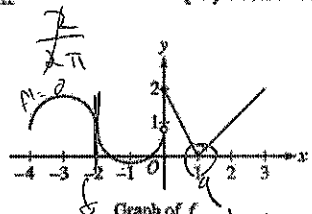
2. $\lim_{x\to\infty} \dfrac{\sqrt{9x^4}+1}{x^2 - 3x + 5}$ is — essentially $3x^2$ $\quad \dfrac{3\cancel{x^2}}{\cancel{x^2}}$

(A) 1

(B) 3

(C) 9

(D) nonexistent



Graph of f

f' undefined $\quad$ sharp point

3. The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

4. An ice sculpture in the form of a sphere melts in such a way that it maintains its spherical shape. The volume of the sphere is decreasing at a constant rate of 2π cubic meters per hour. At what rate, in square meters per hour, is the surface area of the sphere decreasing at the moment when the radius is 5 meters? (Note: For a sphere of radius r, the surface area is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.)

(A) $\dfrac{4\pi}{5}$

(B) 40π

(C) $80\pi^2$

(D) 100π

$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = -2\pi$ $\quad$ Find $\frac{dA}{dt}$ when $r = 5$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-2\cancel{\pi} = 4\cancel{\pi}(5)^2 \frac{dr}{dt}$

$\frac{-2}{100} = \frac{dr}{dt}$

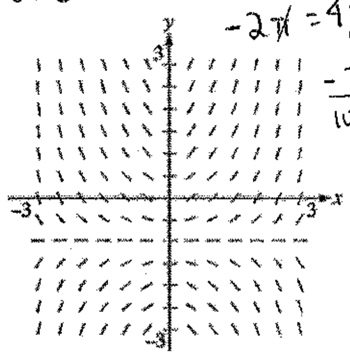
$\frac{dr}{dt} = \frac{-1}{50}$

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(5)\left(\frac{-1}{50}\right)$

$= \frac{-40\pi}{50}$



5. Shown above is a slope field for which of the following differential equations?

(A) $\dfrac{dy}{dx} = xy + x = x(y+1)$

(B) $\dfrac{dy}{dx} = xy + y$

(C) $\dfrac{dy}{dx} = y + 1$

(D) $\dfrac{dy}{dx} = (x+1)^2$

$y = -1 \to \frac{dy}{dx} = 0$

$x = 0 \to \frac{dy}{dx} = 0$

$f(x) = \begin{cases} 2x - 2 & \text{for } x < 3 \\ 2x - 4 & \text{for } x \geq 3 \end{cases}$ $\quad f'(x) = \begin{cases} 2 & x < 3 \\ 2 & x > 3 \end{cases}$

6. Let f be the piecewise-linear function defined above. Which of the following statements are true?

I. $\lim_{h\to 0^-} \dfrac{f(3+h) - f(3)}{h} = 2$ — would come from $2x-2$ — would come from $2x-4$, contradicts

II. $\lim_{h\to 0^+} \dfrac{f(3+h) - f(3)}{h} = 2$ $\quad f'(3) = 2$ from right

III. $f'(3) = 2$ $\quad$ function not cont.

(A) None

(B) II only

(C) I and II only

(D) I, II, and III

7. If $f(x) = \int_1^{x^3} \dfrac{1}{1 + \ln t}\, dt$ for $x \geq 1$, then $f'(2) =$

(A) $\dfrac{1}{1 + \ln 2}$

(B) $\dfrac{12}{1 + \ln 2}$

(C) $\dfrac{1}{1 + \ln 8}$

(D) $\dfrac{12}{1 + \ln 8}$

$f'(x) = \dfrac{1}{1 + \ln(x^3)} \cdot 3x^2$

$f'(2) = \dfrac{1}{1 + \ln 8} \cdot 3(4)$

$= \dfrac{12}{1 + \ln 8}$

8. Which of the following limits is equal to $\int_3^5 x^4\, dx$?

(A) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{k}{n}\right)^4 \frac{1}{n}$

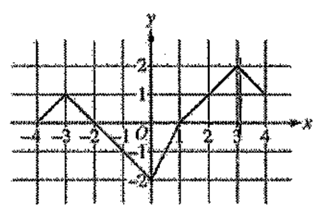
(B) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{k}{n}\right)^4 \frac{2}{n}$

(C) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{2k}{n}\right)^4 \frac{1}{n}$

(D) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{2k}{n}\right)^4 \frac{2}{n}$

$\Delta x = \dfrac{5-3}{n} = \dfrac{2}{n}$

right riem sum adding $\frac{2}{n}$ each time $\left(3 + \frac{2k}{n}\right)$



Graph of f

9. The function f is continuous for $-4 \leq x \leq 4$. The graph of f shown above consists of five line segments. What is the average value of f on the interval $-4 \leq x \leq 4$?

(A) $\dfrac{1}{8}$

(B) $\dfrac{3}{16}$

(C) $\dfrac{15}{16}$

(D) $\dfrac{3}{2}$

$\dfrac{1}{4-(-4)} \int_{-4}^{4} f(x)$

$\dfrac{1}{8}\left(\frac{1}{2}(2)(1) - \frac{1}{2}(3)(2) + \frac{1}{2}(2)(2) + \frac{1}{2}(2 \cdot 1)\right)$

$\dfrac{1}{8}\left(1 - 3 + 2 + \frac{3}{2}\right)$

$\dfrac{1}{8}\left(\frac{3}{2}\right) = \dfrac{3}{16}$

t	0	2
$f(t)$	4	12

$|y| = e^{kt+\ln 4}$

$|y| = 4e^{kt}$

10. Let $y = f(t)$ be a solution to the differential equation $\frac{dy}{dt} = ky$, where k is a constant. Values of f for selected values of t are given in the table above. Which of the following is an expression for $f(t)$?

(A) $4e^{\frac{t}{2}\ln 3}$

(B) $e^{\frac{t}{2}\ln 9} + 3$

(C) $2t^2 + 4$

(D) $4t + 4$

$\frac{dy}{y} = k\,dt$

$\ln|y| = kt + C$

$\ln|4| = 0 + C$

$\ln|y| = kt + \ln 4$

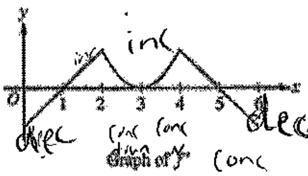
$12 = 4e^{k(2)}$

$3 = e^{2k}$

$\ln 3 = 2k$

$k = \frac{\ln 3}{2}$

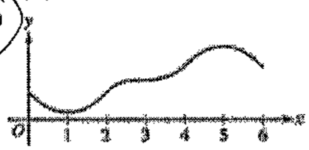
Calculator Section (Some of the following questions require a calculator)



Graph of f'

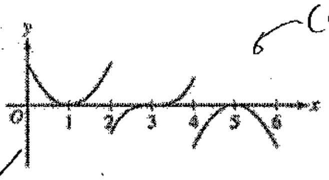
11. The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f?

(A)



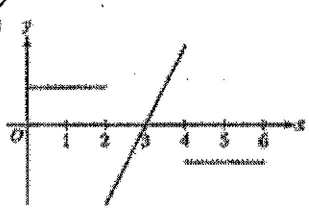
(B)

(C)



can't be since $f'(2)$ & $f'(4)$ wouldn't exist.

(D)



12. The derivative of a function f is given by $f'(x) = e^{\sin x} - \cos x - 1$ for $0 < x < 9$. On what intervals is f decreasing?

(A) $0 < x < 0.633$ and $4.115 < x < 6.916$

(B) $0 < x < 1.947$ and $5.744 < x < 8.230$

(C) $0.633 < x < 4.115$ and $6.916 < x < 9$

(D) $1.947 < x < 5.744$ and $8.230 < x < 9$

f' neg

13. The temperature of a room, in degrees Fahrenheit, is modeled by H, a differentiable function of the number of minutes after the thermostat is adjusted. Of the following, which is the best interpretation of $H'(5) = 2$?

rate of time

(A) The temperature of the room is 2 degrees Fahrenheit, 5 minutes after the thermostat is adjusted.

(B) The temperature of the room increases by 2 degrees Fahrenheit during the first 5 minutes after the thermostat is adjusted.

(C) The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Fahrenheit per minute.

(D) The temperature of the room is increasing at a rate of 2 degrees Fahrenheit per minute, 5 minutes after the thermostat is adjusted.

14. A function f is continuous on the closed interval $[2, 5]$ with $f(2) = 17$ and $f(5) = 17$. Which of the following additional conditions guarantees that there is a number c in the open interval $(2, 5)$ such that $f'(c) = 0$?

(A) No additional conditions are necessary.

(B) f has a relative extremum on the open interval $(2, 5)$.

(C) f is differentiable on the open interval $(2, 5)$. MVT

(D) $\int_2^5 f(x)\,dx$ exists.

15. A rain barrel collects water off the roof of a house during three hours of heavy rainfall. The height of the water in the barrel increases at the rate of $r(t) = 4t^3e^{-1.5t}$ feet per hour, where t is the time in hours since the rain began. At time $t = 1$ hour, the height of the water is 0.75 foot. What is the height of the water in the barrel at time $t = 2$ hours?

(A) 1.361 ft

(B) 1.500 ft

(C) 1.672 ft

(D) 2.111 ft

height $= \int r(t)$

$h(2) = h(1) + \int_1^2 4t^3e^{-1.5t}\,dt$

$= .75 + 1.361 = 2.111$

16. A race car is traveling on a straight track at a velocity of 80 meters per second when the brakes are applied at time $t = 0$ seconds. From time $t = 0$ to the moment the race car stops, the acceleration of the race car is given by $a(t) = -6t^2 - t$ meters per second per second. During this time period, how far does the race car travel?

(A) 188.229 m

(B) 198.766 m

(C) 260.042 m

(D) 267.089 m

$v(t) = \int a(t) = \int -6t^2 - t\,dt$

$= -2t^3 - \frac{t^2}{2} + C$

$C = 80$

$v(t) = -2t^3 - \frac{t^2}{2} + 80$

$v(t) = 0$ at $t = 3.33862$

$\int_0^{3.33862} |v(t)|\,dt$

$= 198.766$