

PE 3

$$1) \quad a) \quad \frac{g'(6) - g'(-4)}{6 - (-4)} = \frac{1 - (-3)}{6 - (-4)} = \frac{4}{10} = \frac{2}{5}$$

+1 ans

b) Method 1

$$\frac{g(6) - g(-4)}{6 - (-4)}$$

$$g(6) = -1$$

$$g(-4) = g(6) + \int_6^{-4} g'(x) dx$$

$$-1 = \int_6^{-4} g'(x) dx$$

* trig areas cancel

$$-1 - \left(-\frac{1}{2} \left(\frac{3}{2} \right) (3) + \frac{1}{2} (1) \left(\frac{1}{2} \right) \right)$$

$$-1 - \left(-\frac{9}{4} + \frac{1}{4} \right)$$

$$-1 + 2 = 1 \quad \text{+1 ans}$$

or Method 2

$$g(6) - g(-4) = \int_{-4}^6 g'(x) dx$$

$$\rightarrow \frac{0 + \left(-\frac{1}{2} \left(\frac{3}{2} \right) (3) + \frac{1}{2} (1) \left(\frac{1}{2} \right) \right)}{10}$$

$$\frac{-\frac{9}{4} + \frac{1}{4}}{10} \quad \text{+1 ans}$$

$$\frac{-2}{10}$$

$$-\frac{1}{5}$$

$$\frac{g(6) - g(-4)}{6 - (-4)} = \frac{-1 - 1}{6 - (-4)} = \frac{-2}{10} = -\frac{1}{5}$$

+1 ans

c) pt:

$$g(2) = g(6) + \int_6^2 g'(x) dx$$

$$-1 - \int_6^2 g'(x) dx$$

$$-1 - \left(\int_2^4 g'(x) dx + \int_4^6 g'(x) dx \right)$$

$$-1 - \left(\int_2^4 3 \cos\left(\frac{\pi}{4}x\right) + \left(\frac{1}{2}(1)(1+3)\right) \right)$$

$$-1 - \left(\left[\frac{12}{\pi} \sin\left(\frac{\pi}{4}x\right) \right]_2^4 - 2 \right)$$

$$-1 - \left(\left(\frac{12}{\pi} \sin(\pi) - \frac{12}{\pi} \sin\left(\frac{\pi}{2}\right) \right) - 2 \right)$$

$$-1 - \left(-\frac{12}{\pi} - 2 \right)$$

$$1 + \frac{12}{\pi}$$

$$\left(2, 1 + \frac{12}{\pi} \right) \quad +1 \text{ } g(2)$$

Slope: $g'(2) = 0$

$$y - \left(1 + \frac{12}{\pi} \right) = 0 \quad (x - 2)$$

+1 tangent equation

d) candidates for abs max value are endpoints: $x = -4, x = 2$ and rel max, $x = 2$ since g' changes from pos to neg

$x \mid g(x)$

$-4 \mid g(-4) = 1$ (from b)

$2 \mid g(2) = 1 + \frac{12}{\pi}$ ← close to 4

+1 $x = 2$ is a rel max

+1 considers endpoints

$12 \mid g(6) + \int_6^{12} g'(x) dx = -1 + \frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) + (1)(3)$ +1 answer.

$-1 + \frac{1}{2} + 2 + 3 = 4.5$ | The abs max value is $1 + \frac{12}{\pi}$

$$e) \lim_{x \rightarrow 8} 2 - \sqrt[3]{x} = 2 - \sqrt[3]{8} = 0$$

Since g is diff, g is cont, so

$$\lim_{x \rightarrow 8} g(x) = g(8) = 0 \quad (\text{work below})$$

+1 verifies conditions and attempts L'Hopital's rule

$$g(8) = g(6) + \int_6^8 g'(x) dx = -1 + \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) = 0$$

By L'Hopital's rule

$$\lim_{x \rightarrow 8} \frac{2 - \sqrt[3]{x}}{g(x)} = \lim_{x \rightarrow 8} \frac{-\frac{1}{3}x^{-\frac{2}{3}}}{g'(x)} = \frac{-\frac{1}{3}(8)^{-\frac{2}{3}}}{g'(8)} = \frac{-\frac{1}{3}(8)^{-\frac{2}{3}}}{1} = -\frac{1}{12}$$

+1 ans

f) There is a pt of inf at $x=0$ & $x=6$ because g' changes from inc to dec.

There is a pt of inf at $x=4$ and $x=7$ because g' changes from dec to inc.

+1 ans w/ just.

$$g) g''(2)$$

$$\text{At } x=2, g''(x) = -3 \sin\left(\frac{\pi}{4}x\right) \frac{\pi}{4}$$

+1 $g''(2)$

$$g''(2) = \left[-\frac{3\pi}{4} \sin\left(\frac{\pi}{4}(2)\right) \right] = -\frac{3\pi}{4}$$

$g''(9) =$ does not exist.

At $x=9$, g' is discontinuous, therefore g' is non diff at $x=9$ +1 ans w/ reason

h) g is dec and conc up on $(-4, -2) \cup (4, 5)$ because g' is neg and inc. +1 ans w/ reason

2

a) $v'(16) = \frac{v(20) - v(12)}{20 - 12} = \frac{240 - 200}{20 - 12} \text{ m/min}^2$ *+1 approx*

Johanna's acceleration is approx. $\frac{40}{8} = 5 \text{ m/min}^2$
 when $t = 16$ minutes. *+1 int. w/ units*

b)

	12	8	4	16
0	12	20	24	40
0	200	240	220	150

\downarrow 220

$12(200) + 8(240) + 4(220) + 16(150)$

7600

+1 Right Riemann w/ unequal width and abs value

c) $B'(t) = 3t^2 - 12t$ *+1 uses B'(t)*

$B'(5) = 3(5)^2 - 12(5) = 75 - 60 = 15$
+1 ans

d) $B'(5) = 15$ pos

$B(5) = 5^3 - 6(5)^2 + 300 = 125 - 150 + 300 = 275$ pos

Bob's speed is inc because velocity & acceleration have the same sign at $t=5$.
+1 ans w/ reason B(t) B'(t)

e) $\frac{B(10) - B(0)}{10 - 0} = \frac{(10^3 - 6(10)^2 + 300) - 300}{10} \text{ m/min}^2 = \frac{1000 - 600}{10} = 40$
+1 avg accel *+1 ans w/ units*

Suggested Scoring:

Raw Score:	Exam Score:
14-23	5
12-13	4
9-11	3
6-8	2
0-5	1

As previously mentioned, College Board has not predetermined the scores needed to earn a 3,4, or 5 for this year. The level of difficulty of the exam will be evaluated with the goal of having scoring distributions to be similar to previous years. However, Q1 will be worth 60% of your overall score and Q2 will be worth 40%. This rubric is just a potential guide and meant to be a helpful tool to gauge your performance. It is not a guarantee of how many points questions will be worth and where the cuts off are.