

## AP Calculus AB 2020 Practice Exam 2

### Reminders:

Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

Calculators are permitted but are not required for any part of any question.

The test is open book/open note, but you may not consult with any other individuals.

### Instructions:

Write your solution using dark pencil or ink on white paper. Label each part of each question. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

All pages need to be labeled with **student AP ID number, student initials, and page number**.

All students will be responsible for watching the timer themselves.

Set a 30 minute timer for Question 1. Practice keeping track of your time. When the timer has 5 minutes left, take a photo of your work and transfer it to the device you are using to access your questions. Do not move on to Question 2 until the time has finished for the first timer. (During the actual test, the second question will not appear until the 30 minutes have ended for Question 1.)

Set a 20 minute timer for Question 2. When the timer has 5 minutes left, take a photo of your work and transfer it to the device you are using to access your questions.

### Structure:

Students must submit on the same device used to access the questions.

I recommend one of the following methods:

1. Access the questions on a laptop, complete the work on white paper, take a photo of the work using your phone, and then send the photo to the laptop (using Airdrop, or email it to yourself)
2. Access the questions on a tablet, complete the work on white paper, then take a photo of your work on the tablet.

**CALCULUS AB**

**Part A**

**Time—25 minutes**

**Number of problems—1**

**A GRAPHING CALCULATOR IS NOT REQUIRED FOR THESE PROBLEMS.**

$t$ (days)	0	3	5	7	9	12	14
$h(t)$ (# of butterflies)	1	2	4	7	11	16	22

1. The number of butterflies spotted in Amelie's garden each afternoon is modeled by a differentiable and increasing function  $h$  for  $0 \leq t \leq 14$ , where  $t$  is measured in days. Selected values of  $h(t)$  are given in the table above.
- (a) Use the data in the table to find an approximation for  $h'(8)$ . Using correct units, interpret the meaning of this value in the context of the problem.
- (b) Explain why there must be at least one time  $t$ , for  $0 < t < 14$ , such that  $h'(t) = 1.5$ .
- (c) Approximate the value of  $\int_0^{14} h(t) dt$  using a left Riemann sum on the subintervals  $[0, 5]$ ,  $[5, 9]$ , and  $[9, 14]$ . Is this approximation greater than or less than  $\int_0^{14} h(t) dt$ ? Give a reason for your answer.
- (d) The number of bees spotted in Amelie's garden during the same 14-day period is modeled by the function  $g(t) = 7e^{\frac{1}{7}(t-1)} + \cos(2t - 16)$  for  $0 \leq t \leq 14$ , where  $t$  is measured in days. At time  $t=8$ , is the number of bees increasing at an increasing rate or increasing at a decreasing rate? Based on your answer, what can you conclude about the graph of  $g(t)$  at  $t=8$ . Justify your answer.
- (e) The number of bees spotted in Amelie's garden can also be modeled by the function  $B(x) = 50\sqrt{k + 2x}$  where  $x$  is the daily high temperature, in degree Fahrenheit, and  $k$  is a positive constant. When the number of bees spotted is 100, the daily high temperature is increasing at a rate of  $2^\circ\text{F}$  per day. According to this model, how quickly is the number of bees changing with respect to time when 100 bees are spotted? Indicate units in your answer.

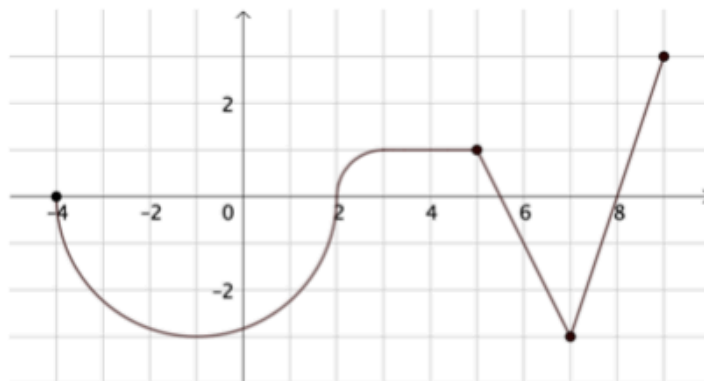
**CALCULUS AB**

**Part B**

**Time—15 minutes**

**Number of problems—1**

**A GRAPHING CALCULATOR IS NOT REQUIRED FOR THESE PROBLEMS.**



2. The function  $f$  is defined on the closed interval  $[-4, 9]$ . The graph of  $f$  consists of a semicircle, a quarter circle, and three linear segments, as shown in the figure above. Let  $g$  be the function defined by  $g(x) = 3x + \int_2^x f(t) dt$ .

(a) Find  $g(9)$  and  $g'(9)$ .

(b) Find the value of  $x$  in the closed interval  $[-4, 9]$  at which  $g$  attains its maximum value. Justify your answer.

(c) Find  $f'(2)$  or state that it does not exist. Justify your answer.

(d) Find  $\lim_{x \rightarrow 3} \left( \frac{8g(2x) - 2\pi - 160}{e^{x-3} - 1} \right)$ .