AP Calculus AB 2020 Practice Exam 1

Reminders:
Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Calculators are permitted but are not required for any part of any question.

The test is open book/open note, but you may not consult with any other individuals.

Students need to submit their work on the same device that is used to access the questions.

Instructions:

Write your solution using dark pencil or ink on white paper. Label each part of each question. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

All students will be responsible for watching the timer themselves.

Set a 30 minute timer for Question 1. Practice keeping track of your time. When the timer has 5 minutes left, take a photo of your work and transfer it to the device you are using to access your questions.

Do not move on to Question 2 until the time has finished for the first timer. (During the actual test, the second question will not appear until the 30 minutes have ended for Question 1.)

Set a 20 minute timer for Question 2. When the timer has 5 minutes left, take a photo of your work and transfer it to the device you are using the access your questions.

CALCULUS AB

Part A

Time—25 minutes

Number of problems—1

A GRAPHING CALCULATOR IS NOT REQUIRED FOR THESE PROBLEMS.

Graph of $v_a(t)$

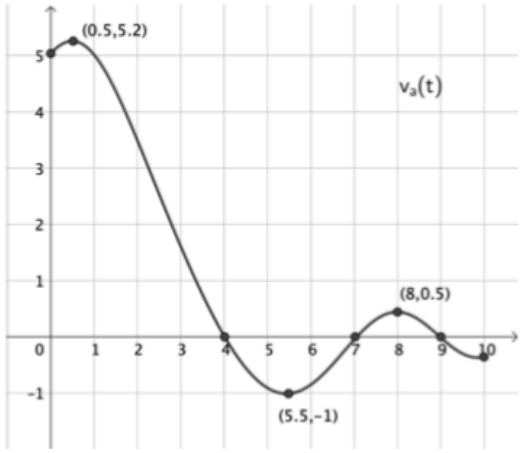


Table 1: Particle B

t	$s_b(t)$	$v_b(t)$
0	2	4
1	8	2
2	12	1
3	15	0
5	13	-2
7	12	-1
8	12	0
10	18	5

1. The velocity of a particle, A, moving along the x-axis is given by the function $v_a(t)$, as shown in the graph above, where $v_a(t)$ is measured in meters per second and t is measured in seconds.

   A second particle, B, also moves along the x-axis so that it's position, $s_b(t)$ meters, and velocity, $v_b(t)$ meters per second, for $0 \le t \le 10$, are differentiable functions with selected values shown in the table above. The velocity, $v_b(t)$ has exactly two zeros on the interval $0 \le t \le 10$.

   (a) Is the speed of Particle A increasing or decreasing at time $t = 6$? Explain.

   (b) For how many values of t, $0 \le t \le 10$, is the speed of Particle A equal to 1 meter per second? Justify your answer.

   (c) Find all times t on the interval $0 \le t \le 10$ at which Particle A changes direction. Justify your answer.

   (d) For $0 \le t \le 10$, must there be a time when $s_b(t) = 7$? Justify your answer.

   (e) Find the average velocity of Particle B on the interval $0 \le t \le 10$. Include units.

   (f) For $0 \le t \le 10$, find all times t during which the two particles travel in the same direction.

   (g) On the interval $[8, 10]$, which is greater: the average acceleration of Particle A or the average acceleration of Particle B? Give a reason for your answer.

   (h) Is there guaranteed a time t on the interval $7 < x < 10$ such that the acceleration of Particle B is equal to 2? Justify your answer.

CALCULUS AB

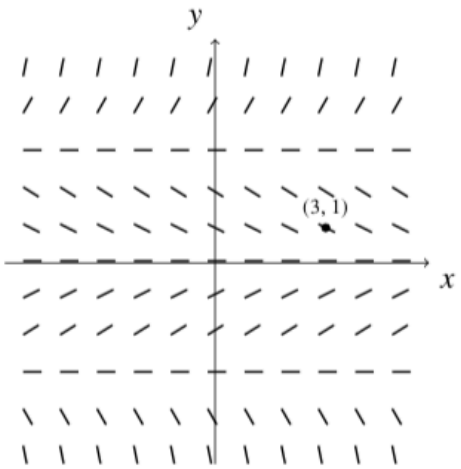
Part B

Time—15 minutes

Number of problems—1

A GRAPHING CALCULATOR IS NOT REQUIRED FOR THESE PROBLEMS.

Slope field of $\frac{dy}{dx} = \frac{1}{16}y(y^2 - 9)$



2. The differential equation $\frac{dy}{dx} = \frac{1}{16}y(y^2 - 9)$ has the slope field shown above. Let $y = f(x)$ be the particular solution to this differential equation that passes through the point $(3, 1)$.

   (a) Use the line tangent to the graph of $y = f(x)$ at $x = 3$ to approximate $f(3.2)$.

   (b) Find $\frac{d^2y}{dx^2}$. Is your answer to part (a) an underestimate or overestimate? Justify your response.

   (c) Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. Justify your answer using the slope field shown above.

(d) Let $y = g(x)$ be the particular solution to the differential equation with the initial condition $g(0) = -2$. Does g have a relative minimum, a relative maximum, or neither at x=0? Justify your answer.