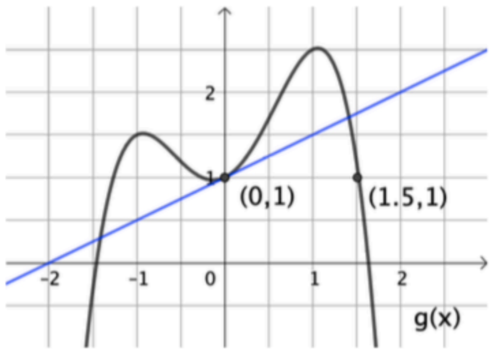
# AP Test Prep Questions, Week 6

All of the following questions are NonCalculator.

## FRQ 1




The graph of $g(x)$ is shown above, along with the line tangent to the graph of $g(x)$ at $x = 0$. Let $f$ be a differentiable function defined by $f(x) = \sqrt{10 - g(x)}$.

(a) Write the equation of the line tangent to $f(x)$ at $x = 0$.

(b) Let $g(2) = a$. Evaluate $\int_0^2 f(x)g'(x)\,dx$. Express your answer in terms of $a$.

(c) Evalute $\lim_{x \to 0} \left( \frac{f(x) - 3}{g(x) - e^{2x}} \right)$, or show that it does not exist.

(d) Is there guaranteed a value $c$ on the interval $0 < x < 1.5$ such that $f'(c) = 0$? Explain.

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## 2007#5

| $t$ (minutes) | 0 | 2 | 5 | 7 | 11 | 12 |
|---|---|---|---|---|---|---|
| $r'(t)$ (feet per minute) | 5.7 | 4.0 | 2.0 | 1.2 | 0.6 | 0.5 |

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function $r$ of time $t$, where $t$ is measured in minutes. For $0 < t < 12$, the graph of $r$ is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius $r$ is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$ ? Give a reason for your answer.

**2002 Form B #4**

The graph of a differentiable function $f$ on the closed interval $[-3,15]$ is shown in the figure above. The graph of $f$ has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

Graph of $f$

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is $g$ decreasing? Justify your answer.

(c) On what intervals is the graph of $g$ concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

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**2005#4**

| $x$ | 0 | $0 < x < 1$ | 1 | $1 < x < 2$ | 2 | $2 < x < 3$ | 3 | $3 < x < 4$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $-1$ | Negative | 0 | Positive | 2 | Positive | 0 | Negative |
| $f'(x)$ | 4 | Positive | 0 | Positive | DNE | Negative | $-3$ | Negative |
| $f''(x)$ | $-2$ | Negative | 0 | Positive | DNE | Negative | 0 | Positive |

Let $f$ be a function that is continuous on the interval $[0, 4)$. The function $f$ is twice differentiable except at $x = 2$. The function $f$ and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of $f$ do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of $x$ at which $f$ has a relative extremum. Determine whether $f$ has a relative maximum or a relative minimum at each of these values. Justify your answer.

(c) Let $g$ be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of $x$ at which $g$ has a relative extremum. Determine whether $g$ has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function $g$ defined in part (c), find all values of $x$, for $0 < x < 4$, at which the graph of $g$ has a point of inflection. Justify your answer.

**FRQ 1**

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(a)

$f(0) = \sqrt{10 - g(0)} = \sqrt{10 - 1} = 3$

$$f'(x) = \frac{-g'(x)}{2\sqrt{10 - g(x)}}$$
$$f'(0) = \frac{-g'(0)}{2\sqrt{10 - g(0)}}$$
$$= -\frac{1}{12}$$

Tangent line equation: $y - 3 = -\frac{1}{12}x$

1: $f'(x)$
1: tangent line equation

(b)

$\int_0^2 f(x)g'(x)\,dx = \int_0^2 \sqrt{10 - g(x)}\,g'(x)\,dx$

Let $u = 10 - g(x)$. Then $du = -g'(x)dx$.

$$\int_0^2 \sqrt{10 - g(x)}\,g'(x)\,dx = -\int_9^{10-a} \sqrt{u}\,du$$
$$= -\frac{2}{3}\left[u^{3/2}\right]_9^{10-a}$$
$$= -\frac{2}{3}[(10 - a)^{3/2} - 27]$$
$$= -\frac{2}{3}(10 - a)^{3/2} + 18$$

1: u-substitution
1: integral
1: answer in terms of $a$

(c)

$\lim_{x\to 0}(f(x) - 3) = [f(0) - 3] = 0$

$\lim_{x\to 0}(g(x) - e^{2x}) = [g(0) - e^0] = 0$

Therefore, L'Hôpital's Rule may be used.

$$\lim_{x\to 0}\left(\frac{f(x) - 3}{g(x) - e^{2x}}\right) = \lim_{x\to 0}\left(\frac{f'(x)}{g'(x) - 2e^{2x}}\right)$$
$$= \frac{f'(0)}{g'(0) - 2}$$
$$= \frac{-\frac{1}{12}}{\frac{1}{2} - 2}$$
$$= \frac{1}{18}$$

1: Verification L'hopital's rule can be used and attempt at derivatives
1: answer

(d) f is diff, so f is cont.

$f(1.5) = \sqrt{10 - g(1.5)} = 3$

$$\frac{f(1.5) - f(0)}{1.5 - 0} = \frac{3 - 3}{1.5} = 0$$

By the MVT, there must be a c in $(0, 1.5)$ such that $f'(c) = 0$.

1:sec slope
1: conc. w/ MVT

**2007#5**

| $t$ (minutes) | 0 | 2 | 5 | 7 | 11 | 12 |
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(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

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| | |
|---|---|
| (a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft<br>Since the graph of $r$ is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$. | $2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$ |
| (b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$<br>$\left.\frac{dV}{dt}\right\vert_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$ | $3 : \begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$ |
| (c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$<br>$= 19.3$ ft<br>$\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes. | $2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$ |
| (d) Since $r$ is concave down, $r'$ is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$. | 1 : conclusion with reason |
| Units of $\text{ft}^3/\text{min}$ in part (b) and ft in part (c) | 1 : units in (b) and (c) |

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| | |
|---|---|
| (a) $g(6) = 5 + \int_6^6 f(t)dt = 5$<br>$g'(6) = f(6) = 3$<br>$g''(6) = f'(6) = 0$ | $3\begin{cases} 1: g(6) \\ 1: g'(6) \\ 1: g''(6) \end{cases}$ |
| (b) $g$ is decreasing on $[-3,0]$ and $[12,15]$ since $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$. | $3\begin{cases} 1: [-3,0] \\ 1: [12,15] \\ 1: \text{justification} \end{cases}$ |
| (c) The graph of $g$ is concave down on $(6,15)$ since $g' = f$ is decreasing on this interval. | $2\begin{cases} 1: \text{interval} \\ 1: \text{justification} \end{cases}$ |
| (d) $\frac{3}{2}(-1 + 2(0+1+3+1+0) - 1)$<br>$= 12$ | 1 : trapezoidal method |

**2005#4**

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Let $f$ be a function that is continuous on the interval $[0, 4)$. The function $f$ is twice differentiable except at $x = 2$. The function $f$ and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of $f$ do not exist at $x = 2$.

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(c) Let $g$ be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of $x$ at which $g$ has a relative extremum. Determine whether $g$ has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function $g$ defined in part (c), find all values of $x$, for $0 < x < 4$, at which the graph of $g$ has a point of inflection. Justify your answer.

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(a) $f$ has a relative maximum at $x = 2$ because $f'$ changes from positive to negative at $x = 2$.

$2: \begin{cases} 1: \text{relative extremum at } x = 2 \\ 1: \text{relative maximum with justification} \end{cases}$

(c) $g'(x) = f(x) = 0$ at $x = 1, 3$.
$g'$ changes from negative to positive at $x = 1$ so $g$ has a relative minimum at $x = 1$. $g'$ changes from positive to negative at $x = 3$ so $g$ has a relative maximum at $x = 3$.

$3: \begin{cases} 1: g'(x) = f(x) \\ 1: \text{critical points} \\ 1: \text{answer with justification} \end{cases}$

(d) The graph of $g$ has a point of inflection at $x = 2$ because $g'' = f'$ changes sign at $x = 2$.

$2: \begin{cases} 1: x = 2 \\ 1: \text{answer with justification} \end{cases}$