

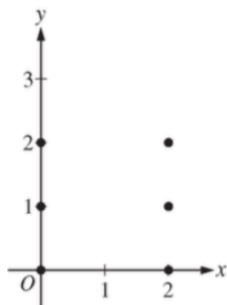
AP Test Prep Questions, Week 5

All of the following questions are NonCalculator.

2016 #4

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$.
Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

2011 #4

The continuous function f is defined on the interval $-4 \leq x \leq 3$.

The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

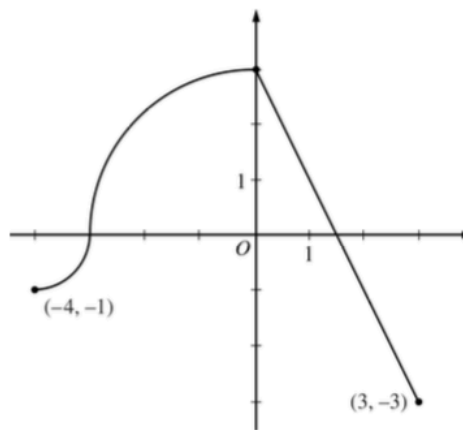
Let $g(x) = 2x + \int_0^x f(t) dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$.
Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change.
Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

2007 #3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.

- (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
- (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.
- (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
- (d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

2013 #3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

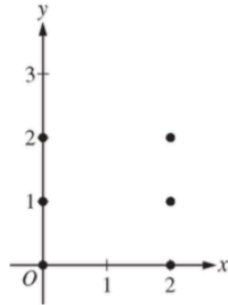
Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^6 C(t) dt$.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

2016 #4

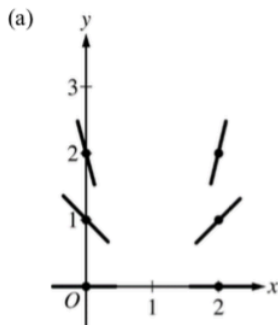
Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.



2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = \frac{3^2}{2-1} = 9$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

An equation for the tangent line is $y = 9(x - 2) + 3$.

$$f(2.1) \approx 9(2.1 - 2) + 3 = 3.9$$

(c) $\frac{1}{y^2} dy = \frac{1}{x-1} dx$
 $\int \frac{1}{y^2} dy = \int \frac{1}{x-1} dx$

$$-\frac{1}{y} = \ln|x-1| + C$$

$$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$$

$$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$$

$$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/5 [1-2-0-0] if no constant of integration

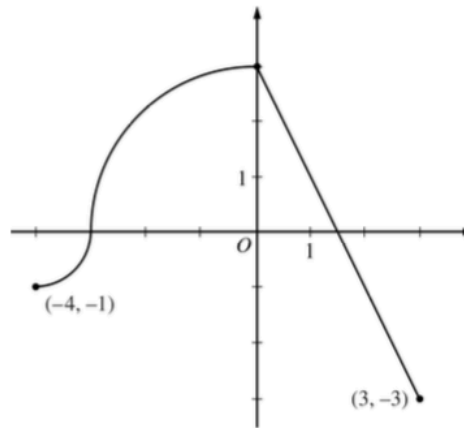
Note: 0/5 if no separation of variables

2011 #4

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Let $g(x) = 2x + \int_0^x f(t) dt$.

- (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
- (b) Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.
- (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t) dt = -6 - \frac{9\pi}{4}$
 $g'(x) = 2 + f(x)$
 $g'(-3) = 2 + f(-3) = 2$

3 : $\begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.
 $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.
 Therefore g has an absolute maximum at $x = \frac{5}{2}$.

3 : $\begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \leq x \leq 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

2 : $\begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

2007 #3

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- (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
- (d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
 $h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$
 Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r , $1 < r < 3$, such that $h(r) = -5$.

(b) $\frac{h(3) - h(1)}{3 - 1} = \frac{-7 - 3}{3 - 1} = -5$
 Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c , $1 < c < 3$, such that $h'(c) = -5$.

(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

(d) $g(1) = 2$, so $g^{-1}(2) = 1$.

$$(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$$

An equation of the tangent line is $y - 1 = \frac{1}{5}(x - 2)$.

2 : $\begin{cases} 1 : h(1) \text{ and } h(3) \\ 1 : \text{conclusion, using IVT} \end{cases}$

2 : $\begin{cases} 1 : \frac{h(3) - h(1)}{3 - 1} \\ 1 : \text{conclusion, using MVT} \end{cases}$

2 : $\begin{cases} 1 : \text{apply chain rule} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : g^{-1}(2) \\ 1 : (g^{-1})'(2) \\ 1 : \text{tangent line equation} \end{cases}$

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(c)

$$2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)$$

1 : midpoint sum

$$2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8$$

$$60.6$$

$$(d) \quad B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$$

$$B'(5) = 6.4e^{-0.4(5)} = \frac{6.4}{e^2} \text{ ounces/min}$$

$$2 : \begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$$