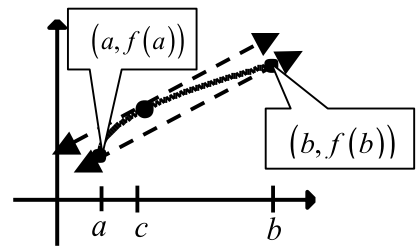
1) Definition of the Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Alternate Form of the Definition: $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

2) Definition of Continuous Functions at Point x=a

1. f(a) exists
2. $\lim_{x \to a} f(x)$ exists. ($\lim_{x \to a-} f(x) = \lim_{x \to a+} f(x)$) (Left hand limit matches right hand limit.)
3. $f(a) = \lim_{x \to a} f(x)$

3) Odd functions: $f(-x) = -f(x)$, Symmetric to origin
Even functions: $f(-x) = f(x)$, Symmetric to y axis.



4) Mean Value Theorem Equation: $f'(c) = \frac{f(b) - f(a)}{b - a}$
for $a < c < b$ (for $f(x)$ continuous on [a, b] and differentiable on (a, b))

(Tangent Slope = Secant Slope)

6) First Fundamental Theorem of Calculus:
$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

7) Second Fundamental Theorem of Calculus:
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

When Chain Rule applies: $\frac{d}{dx}\int_a^u f(t)\,dt = f(u)\frac{du}{dx}$

8) Approximation of the Area Under a Curve

a) **Riemann Sum:** Draw a figure and add up the rectangle areas (left, right, midpoint) (with equal width) width= (b-a)/# of intervals

b) **Trapezoids:** (with unequal widths) add up areas of all trapezoids $A = \frac{1}{2}h(b_1 + b_2)$

(with n equal widths) $A \approx \frac{1}{2}\frac{b-a}{n}\left(f(x_1) + 2f(x_2) + 2f(x_3) + \ldots + 2f(x_n) + f(x_{n+1})\right)$

12) Growth and Decay Equation
$y = Ce^{kt}$ or $y = y_o e^{kt}$

13) Trig Limits

$\lim_{x\to 0} \frac{sinx}{x} = 1$

$\lim\limits_{x\to 0} \frac{x}{sinx} = 1$

$\lim_{h\to 0} \frac{cosh-1}{h}$ =0

** **Remember that we can now use L'Hopital's rule for these limits too**

14) Derivative of an Inverse

$g'(y) = \frac{1}{f'(g(y))} = \frac{1}{f'(x)}$ (Easier to just make a table. Image points have reciprocal slopes.)

15) Finding Asymptotes

Horizontal Asymptotes: **B**igger **o**n **B**ottom **0**, **B**igger **o**n **T**op **N**one, **E**xponents **a**re **t**he **s**ame **D**ivide **C**oefficients. Can also use this for limits toward infinity.
Vertical Asymptotes: Factor Denominator and set equal to zero.

16) Finding Zeros

Set numerator equal to zero

17) Position – Velocity – Acceleration

$s(t)$ or $x(t)$ – $v(t)$ – $a(t)$

$\frac{d}{dt}s(t) = v(t)$ $\qquad \int a(t)dt = v(t) + C$

$\frac{d}{dt}v(t) = a(t)$ $\qquad \int v(t)dt = s(t) + C$

$v(t) = 0$ implies particle at rest.
$v(t) > 0$ implies particle moving to right.
$v(t) < 0$ implies particle moving to left.
Speed = $|v(t)|$

Particle speeds up if velocity and acceleration have the same sign. Particle slows down if velocity and acceleration have opposite signs.

18) Intermediate Value Theorem

If f is continuous on a closed interval [a,b] and c is any number between f(a) and f(b) then there is at least one number x in the interval [a,b] such that f(x)=c

19) Trig Identities

$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$

20) Average Rate of Change vs. Instantaneous Rate of Change

Average rate of change of a Function on [a,b]: $m = \frac{f(b) - f(a)}{b - a}$ (this is just slope between two points)

Instantaneous Rate of Change: use the derivative (slope at <u>one</u> point)

21) Tangent – Normal Lines

Tangent: $y - y_1 = m(x - x_1)$ use the derivative to find m

Normal: $y - y_1 = \frac{-1}{m}(x - x_1)$

22) Curve Sketching for $y = f(x)$:

	f '(x)	f ''(x)
Increasing		
Decreasing		
Concave Up		
Concave Down		

Max. or Min. Points: Relative: $f'(x) = 0$ or $f'(x)$ undefined. **Make an f' number line.**

Absolute:

Extreme Value Theorem: If a function is continuous on a closed interval [a,b], then f has both a max value and a min value on [a,b].

The candidates are relative extrema **or endpoints** of a closed interval.

Justifications for Relative Max: f' changes from positive to negative
Justifications for Relative Min: f' changes from negative to positive

Points of Inflection: $f''(x) = 0$ or $f''(x)$ undefined **Make an f'' number line.**
Justifications for Points of Inflection: f' changes from decreasing to increasing, f' changes from increasing to decreasing, f'' changes from positive to negative or negative to positive.

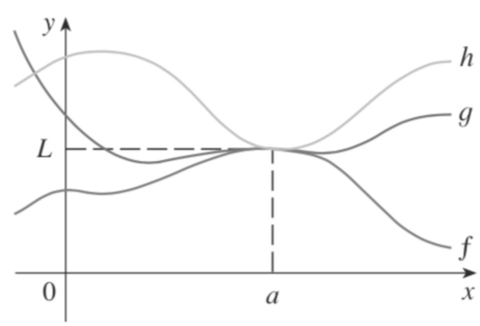
23) Squeeze Theorem

The Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a)
and

$$\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$$

then $\lim_{x\to a} g(x) = L$



24) Derivatives to Memorize

Product Rule: $\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + g'(x)f(x)$

Quotient Rule $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

Chain Rule $\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$

Power Rule: $\frac{d}{dx} x^n = nx^{n-1}$

Log Rules: $\frac{d}{dx} \ln x = \frac{1}{x}$ $\quad \frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

Exponential Rules: $\frac{d}{dx} e^x = e^x$ $\quad \frac{d}{dx} a^x = a^x \ln a$

Trig Rules: $\frac{d}{dx}\sin x = \cos x$ $\quad$ $\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad$ $\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$ $\quad$ $\frac{d}{dx}\csc x = -\csc x \cot x$

Inverse Trig Rules: $\frac{d}{dx}[sin^{-1}x] = \frac{1}{\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}[cos^{-1}x] = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[tan^{-1}x] = \frac{1}{1+x^2}$ $\quad$ $\frac{d}{dx}[cot^{-1}x] = \frac{-1}{1+x^2}$

$\frac{d}{dx}[sec^{-1}x] = \frac{1}{x\sqrt{x^2-1}}$ $\quad$ $\frac{d}{dx}[csc^{-1}x] = \frac{-1}{x\sqrt{x^2-1}}$

** IMPLICIT DIFFERENTIATION can be used when it is inconvenient to solve for y.

25) Integrals to Memorize:
****Use u-substitution if x is not a singular variable.**

Power Rule $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$

Log Rules $\quad$ $\int \frac{1}{x} dx = \ln|x| + C$

Exponential Rules $\quad$ $\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$

Trig Rules

$\int \cos x \, dx = \sin x + C$

$\int \sin x \, dx = -\cos x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \sec x \tan x \, dx = \sec x + C$

$\int \csc^2 x \, dx = -\cot x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

Inverse Trig Rules $\quad$ $\int \frac{1}{\sqrt{1-x^2}} dx = sin^{-1}x + c$ $\quad$ $\int \frac{1}{\sqrt{a^2-x^2}} dx = sin^{-1}\frac{x}{a} + c$

$\int \frac{1}{1+x^2} dx = tan^{-1}x + c$ $\quad$ $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} tan^{-1}\frac{x}{a} + c$

$\int \frac{1}{x\sqrt{x^2-1}} dx = sec^{-1}x + c$ $\quad$ $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} sec^{-1}\frac{x}{a} + c$

26) Geometry Formulas:

Triangles
-Know how to set up proportions using similar triangles
-Area: ½ bh
-Area of Equilateral Triangle: $\frac{s^2\sqrt{3}}{4}$

Rectangles
-Area: $A = l \cdot w$
-Perimeter: $P = 2l + 2w$

Circles
-Area: $A = \pi r^2$
-Relationship between radius and diameter: 2r=d
-Circumference: $C = d\pi$

Rectangular Prism/Box
-Volume: $V = l \cdot w \cdot h$
-Surface Area: $SA = 2(lw + lh + hw)$

Cylinder
-Volume: $V = \pi r^2 h$

Sphere
-Volume: $V = \frac{4}{3}\pi r^3$
-Surface Area: $SA = 4\pi r^2$

Cone
-Volume: $V = \frac{\pi}{3} r^2 h$