College Board Mock Exam #2

Question 1

(a) $g'(5) = \boxed{-\frac{5}{3}}$ +1 Answer

(b) $b'(x) = 4x\,g(x) + 2x^2 g'(x)$ +1 product rule

$b'(5) = 4(5)g(5) + 2(5)^2 g'(5)$

$= \boxed{4(5)(1) + 2(5)^2\left(-\frac{5}{3}\right)}$ +1 answer

(c) $w'(x) = \dfrac{(2x+1)(3h'(x) - 1) - (3h(x) - x)(2)}{(2x+1)^2}$ +1 quotient rule

$w'(5) = \boxed{\dfrac{(2(5)+1)(3(-\frac{5}{3}) - 1) - (3(1) - 5)(2)}{(2(5)+1)^2}}$ +1 answer

(d) $M(x) = g(2x)(2)$ +1 $M'(x)$ $M'(x) = g'(2x)(4)$

$M'(2.5) = g'(5)(4) = \boxed{-\frac{5}{3}(4)}$ +1 answer

(e) +1 secant slope

$\dfrac{2g(8) - 2g(2)}{4-1} = -\frac{5}{3}(4)$; $\dfrac{2(g(8) - g(2))}{3} = -\frac{5}{3}(4)$; $2(g(8) - g(2)) = -20$

$g(8) - g(2) = \boxed{-10}$ +1 answer

(f) since g and f are twice-differentiable, g, g', f, f' are continuous.

so $\lim_{x\to 5} g(x) = g(5)$, $\lim_{x\to 5} g'(x) = g'(5)$, $\lim_{x\to 5} f(x) = f(5)$, $\lim_{x\to 5} f'(x) = f'(5)$.

so $\lim_{x\to 5} \dfrac{x + 5\cos(\frac{1}{5}\pi x)}{3 - \sqrt{f(x)}} = 1$ +1 connecting continuity to limits

since L'Hopital's Rule applies and $\lim_{x\to 5} x + 5\cos(\frac{1}{5}\pi x) = 0$,

$\lim_{x\to 5} 3 - \sqrt{f(x)} = 0$. so $3 - \sqrt{f(5)} = 0$ $-\sqrt{f(5)} = -3$ $\boxed{f(5) = 9}$ +1 $f(5)$

since L'Hopital's Rule applies, +1 Derivatives

$\lim_{x\to 5} \dfrac{1 - \pi\sin(\frac{1}{5}\pi x)}{-\frac{1}{2}(f(x))^{-1/2} \cdot f'(x)} = 1$ $\dfrac{1 - \pi\sin\pi}{-\frac{1}{2}\cdot\frac{1}{\sqrt{9}}\cdot f'(5)} = 1$ $\dfrac{1}{-\frac{1}{6}\cdot f'(5)} = 1$

+1 $f'(5)$ $\boxed{f'(5) = -6}$

9) since $h(x) \le k(x) \le g(x)$ and $h(5) = g(5) = 1$

$k(5) = 1$

since g and h are differentiable, g and h are continuous.

so $\lim_{x \to 5} g(x) = g(5) = 1$ and $\lim_{x \to 5} h(x) = h(5) = 1$.

By the squeeze Theorem, $\lim_{x \to 5} k(x) = 1$

since $\lim_{x \to 5} k(x) = k(5) = 1$, $k(x)$ is continuous when $x = 5$.

+1 continuous w/ Justification that involves squeeze Theorem

Question #2

(a) $f'(x) = \pi \cos \pi x - \dfrac{1}{2-x}$ +1 $f'(x)$

$f'(1) = \pi \cos \pi - 1 = \boxed{-\pi - 1}$ +1 answer

(b) $k'(x) = h'(f(x)+2) \cdot f'(x)$ +1 $k'(x)$ $f(1) = \sin \pi + \ln 1 = 0$

$k'(1) = h'(f(1)+2) \cdot f'(1)$

$= h'(0+2) \cdot (-\pi - 1)$

$= h'(2) \cdot (-\pi - 1) = \boxed{-\frac{1}{3}(-\pi - 1)}$ +1 answer

(c) $\displaystyle\int_{-5}^{-1} g'(x)\,dx = g(x)\Big|_{-5}^{-1} = g(-1) - g(-5) = \boxed{1 - 10}$

+1 answer

(d) $\displaystyle\int_{-1}^{4} h(x)\,dx = \boxed{\frac{1}{2}(1)(1) - \frac{1}{2}(3)(1)}$

2d) I will talk about this question during the Monday video sessions.

+1 integral +1 answer

(e) since g is twice-differentiable, g and g' are continuous.
since $g'(-4) = -1$ and $g'(-3) = 4$ and $-1 < 0 < 4$,
and since $g'(-2) = 1$ and $g'(-1) = -2$ and $-2 < 0 < 1$,
the IVT guarantees a value of c where $-4 < c < -3$ and $g'(c) = 0$
and another value k where $-2 < k < -1$
and $g'(k) = 0$.

so g must have at least 2 horizontal tangents on $-5 < x < 0$.

+1 using IVT after checking conditions and intervals

+1 answer w/ Justification

Suggested Scoring:

Raw Score:	Exam Score:
14-23	5
12-13	4
9-11	3
6-8	2
0-5	1

As previously mentioned, College Board has not predetermined the scores needed to earn a 3,4, or 5 for this year. Instead, they will curve the scores to match the percentages of previous years. However, Q1 will be worth 60% of your overall score and Q2 will be worth 40%. This rubric is just meant to be a helpful tool to gauge your performance.