College Board Mock Exam #1

Question 1

(a) $f'(t)V_p(t) + f(t)V_p'(t)$

$f'(1)V_p(1) + f(1)V_p'(1) = \boxed{2(-29) + 1(-10)}$ +1 answer w/ work

(b) $\frac{1}{2}(.3)(0+55) + \frac{1}{2}(.7)(55+-29) + \frac{1}{2}(1.8)(-29+55)$ +1 trapezoidal sum

(c) $\int_{-6}^{5} f(t)dt = \int_{-6}^{-2} f(t)dt + \int_{-2}^{5} f(t)dt$ +1 Break larger integral into pieces

+1 area under curve for $\int_{-2}^{5} f(t)dt$

$7 = \int_{-6}^{-2} f(t)dt + \frac{1}{2}(1+3)(1) + 3(3) - \frac{1}{4}\pi(3)^2$

$\int_{-6}^{-2} f(t)dt = \boxed{7 - \left(\frac{1}{2}(1+3) + 3(3) - \frac{1}{4}\pi(3)^2\right)}$ +1 answer

(d) $\int_{3}^{5} 2f'(t) + 4\, dt = 2f(t) + 4t \Big|_3^5$ +1 1st Fundamental Thereom applied

$= 2f(5) + 4(5) - 2f(3) - 4(3)$

$= \boxed{0 + 20 - 2(3-\sqrt{5}) - 12}$ +1 answer

(e) $g(t) = \int_{-2}^{t} f(x)dx \Rightarrow g'(t) = f(t)$ +1 $g'(t) = f(t)$

$f(t)$ changes from pos to neg when $x = -1 \Rightarrow$ relative max.

$g(-1) = \int_{-2}^{-1} f(t)dt = -\frac{1}{2}(1)(1) = -\frac{1}{2}$ (answer is negative area since bounds are backwards)

+1 considers $x = -1$ as candidate

$g(-2) = \int_{-2}^{-2} f(t)dt = 0$

$g(5) = 11 - \frac{9\pi}{4}$

$\boxed{\text{Abs max of } g \text{ is } 11 - \frac{9\pi}{4} \text{ on } [-2, 5]}$

+1 answer w/ analysis

(f) $g'(t) = f(t)$

$g''(t) = f'(t)$ +1 $g''(t) = f'(t)$

The rate of change of g is decreasing when $t=3$ since f is decreasing, $f'(3)$ is negative

+1 answer w/ reason

(g) $\lim_{t \to 1} \frac{e^t - 3f(t)}{V_p(t) - \cos \pi t}$

since $V_p(t)$ is differentiable, $V_p(t)$ is continuous. so $\lim_{t \to 1} V_p(t) = V_p(1)$.

+1 description of $V_p(t)$ being continuous

$= \frac{e^1 - 3f(1)}{V_p(1) - \cos \pi} = \frac{e-3}{-29+1} = \boxed{\frac{e-3}{-28}}$

+1 answer

$\boxed{\text{Question \#2}}$

ⓐ $\frac{dV}{dt} = 2\pi r \cdot \frac{dr}{dt} h + \pi r^2 \cdot \frac{dh}{dt}$

$\frac{dr}{dt} = 0$ since r is constant.

$\frac{dV}{dt} = \boxed{\pi(3)^2\left(-\frac{1}{5}\sqrt{10}\right) \text{ m}^3/\text{s}}$

+1 $\frac{dV}{dt}$ expression

+1 answer w/ units

ⓑ $\frac{d^2h}{dt^2} = -\frac{1}{5} \cdot \frac{1}{2} h^{-1/2} \cdot \frac{dh}{dt}$

+2 derivative w/ chain and implicit

$= -\frac{1}{10} \cdot \frac{1}{\sqrt{h}} \cdot \left(-\frac{1}{5}\sqrt{h}\right)$

$= \frac{1}{50} \cdot \frac{1}{\sqrt{8}} \cdot \sqrt{8} = \frac{1}{50}$

Rate of change of height is increasing since $\frac{d^2h}{dt^2}$ is positive

+1 answer w/ reason

ⓒ $\frac{dh}{\sqrt{h}} = -\frac{1}{5} dt$ +1 separate

$\int h^{-1/2} \, dh = \int -\frac{1}{5} \, dt$

+1 antiderivatives

$2h^{1/2} = -\frac{1}{5} t + C$

$2\sqrt{16} = -\frac{1}{5}(0) + C$ +1 for +C and using initial condition

$C = 8$

$2h^{1/2} = -\frac{1}{5} t + 8$

$h^{1/2} = -\frac{1}{10} t + 4$

$\boxed{h = \left(-\frac{1}{10} t + 4\right)^2}$ +1 h isolated

Note: Max 0/4 if no separation. and Max 2/4 if no +C.

Suggested Scoring:

Raw Score:	Exam Score:
14-23	5
12-13	4
9-11	3
6-8	2
0-5	1

As previously mentioned, College Board has not predetermined the scores needed to earn a 3,4, or 5 for this year. Instead, they will curve the scores to match the percentages of previous years. However, Q1 will be worth 60% of your overall score and Q2 will be worth 40%. This rubric is just meant to be a helpful tool to gauge your performance.