A particle moves along the x-axis. The velocity of the particle is modeled by a strictly decreasing, twice differentiable function $v(t)$ measured in meters per second. Select values of $v(t)$ at specific times t, measured in seconds, are given below. It is known at time $t = 7$, the particle's position is 3 units to the right of the origin.

t (seconds)	2	3	5	7	9
$v(t)$ (meters per second)	3	1	0	-6	-8

(a) Estimate $v'(2.5)$ and $v'(6)$. Interpret the meanings in context including units.

(b) State whether the particle is speeding up or slowing down at both $t = 2.5$ and $t = 6$.

(c) The particle's position is modeled by the function $P(t)$. Write an equation of the tangent line to the graph of P at $t = 7$. Use the tangent line to approximate $P(8)$.

(d) Is the estimate in part (c) an under approximation or over approximation of $P(8)$? Explain how you know.

(e) Claire, a calculus student, uses a left Riemann sum of three subintervals to approximate $\int_2^7 v(t)\,dt$. Is her approximation an overestimate or underestimate of the actual value? Explain how you know.

(f) Another particle Q is also moving along the x-axis. Let $Q(x) = 4 + 5x - x^2$. State open interval(s) during $2 \leq t \leq 9$ when particle P and particle Q are moving in the same direction.

Answer Key to AP Live Last Review

(a) $v'(2.5) \approx \frac{v(3)-v(2)}{3-2} = \frac{1-3}{1} = -2\ m/s^2$

At 2.5 seconds, the particle's velocity is decreasing at a rate of $2\ m/s^2$.

$v'(6) \approx \frac{v(7)-v(5)}{7-5} = \frac{-6-0}{2} = -3\ m/s^2$

At 6 seconds, the particle's velocity is decreasing at a rate of $3\ m/s^2$.

(b) since $v(t)$ is strictly decreasing, $a(t)$ is always negative. since $v(t)$ is strictly decreasing and differentiable, Also, if $v(t)$ is strictly decreasing, then $v(3) < v(2.5) < v(2)$. so $1 < v(2.5) < 3$. so $v(2.5)$ is positive. so particle is slowing down when $t = 2.5$ since $v(2.5)$ and $a(2.5)$ are opposite signs.

similarly, $v(7) < v(6) < v(5)$, so $-6 < v(6) < 0$, so $v(6)$ is negative. so particle is speeding up when $t = 6$ since $v(6)$ and $a(6)$ are the same sign.

(c) $y - 3 = -6(x - 7)$

$y - 3 = -6(8 - 7)$

$P(8) \approx \boxed{-6 + 3}$

(d) The approximation in part C is an **overestimate**

since $v(t)$ is strictly decreasing, $a(t)$ is strictly negative, so $P(t)$ is concave down.

(e) Her Riemann sum would be an overestimate since $v(t)$ is strictly decreasing and she's using a left-hand sum.

Note: the question did not ask us to find the sum. If it did, the answer/work would be:

$$1(3) + 2(1) + 2(0)$$

(f) $Q(x) = 4 + 5x - x^2$

$Q'(x) = 5 - 2x$

$0 = 5 - 2x$

$x = 5/2$

$(2, 5/2) \cup (5, 9)$ since $v(t)$ and $Q'(x)$ have the same signs