Unit 6 AP Prep Questions

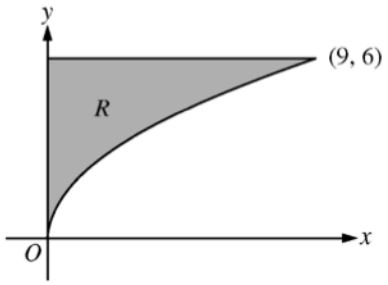
**All of the following questions should be completed with all work shown and correct answers. All AP questions should be scored using the Collegeboard Rubric in a different colored pen.**

19. FR: Noncalc Area & Volume (2010, 1999, 2009, 2013)
20. FR: Calc Area & Volume (2005, 2005, 2003)
21. Unit 6 Review Sheet
22. FR: Noncalc Area & Volume (2019)
23. Unit 6 Take Home Quiz
24. FR: Calc Area & Volume (2011) *(Will be passed out Tuesday 1/28 or Wednesday 1/29)*

**Solutions and Scoring Rubric for FR Questions:**

AP Questions are listed below in the order of the Table of Contents

**2010**




Let $R$ be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the $y$-axis, as shown in the figure above.

(a) Find the area of $R$.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 7$.

(c) Region $R$ is the base of a solid. For each $y$, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the $y$-axis is a rectangle whose height is 3 times the length of its base in region $R$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

| **Answers** | **Scoring Rubric** |
|---|---|
| (a) 18 | (a) $3: \begin{cases} 1: \text{integrand} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$ |
| (b) $\pi\int_0^9 \left((7-2\sqrt{x})^2 - (7-6)^2\right) dx$ | (b) $3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits and constant} \end{cases}$ |
| (c) $\int_0^6 3\left(\frac{y^2}{4}\right)^2 dy$ | (c) $3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$ |

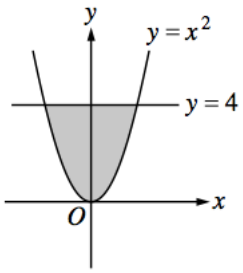
## 1999

The shaded region, $R$, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of $R$.

(b) Find the volume of the solid generated by revolving $R$ about the $x$–axis.

(c) There exists a number $k$, $k > 4$, such that when $R$ is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of $k$.




| Answers | Scoring Rubric |
|---|---|
| (a) $\frac{32}{3}$ | (a) **2** { 1: integral; 1: answer } |
| (b) $\frac{256}{5}\pi$ | (b) **3** { 1: limits and constant; 1: integrand; 1: answer } |
| (c) $\pi \int_{-2}^{2} \left[(k - x^2)^2 - (k - 4)^2\right] dx = \frac{256\pi}{5}$ | (c) **4** { 1: limits and constant; 2: integrand <−1> each error; 1: equation } |

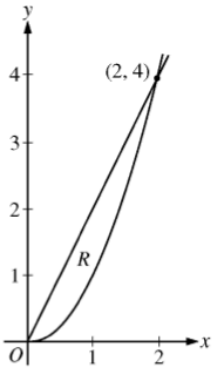
## 2009

Let $R$ be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(a) Find the area of $R$.

(b) The region $R$ is the base of a solid. For this solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

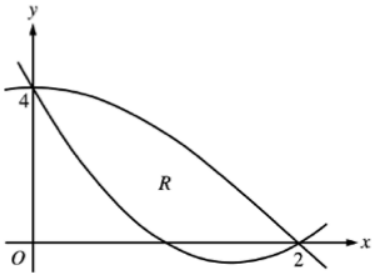
(c) Another solid has the same base $R$. For this solid, the cross sections perpendicular to the $y$-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.




| Answers | Scoring Rubric |
|---|---|
| (a) $\frac{4}{3}$ | (a) 3 : { 1 : integrand; 1 : antiderivative; 1 : answer } |
| (b) $\frac{4}{\pi}$ | (b) 3 : { 1 : integrand; 1 : antiderivative; 1 : answer } |
| (c) $\int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$ | (c) 3 : { 2 : integrand; 1 : limits } |

## 2013

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let $R$ be the region bounded by the graphs of $f$ and $g$, as shown in the figure above.




(a) Find the area of $R$.

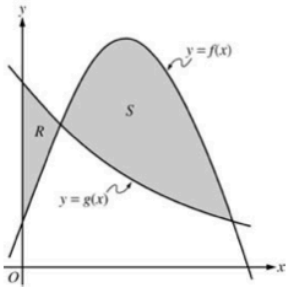
(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 4$.

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

| Answers | Scoring Rubric |
|---|---|
| (a) $\frac{16}{\pi} - \frac{4}{3}$<br><br>(b)<br>$\pi\int_0^2\left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$<br>(c)<br>$\int_0^2[g(x) - f(x)]^2\, dx$ | (a)<br>4 : { 1 : integrand; 2 : antiderivative; 1 : answer }<br>(b)<br>3 : { 2 : integrand; 1 : limits and constant }<br>(c)<br>2 : { 1 : integrand; 1 : limits and constant } |

## 2005 (Calculator)

Let $f$ and $g$ be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let $R$ be the shaded region in the first quadrant enclosed by the $y$-axis and the graphs of $f$ and $g$, and let $S$ be the shaded region in the first quadrant enclosed by the graphs of $f$ and $g$, as shown in the figure above.




(a) Find the area of $R$.

(b) Find the area of $S$.

(c) Find the volume of the solid generated when $S$ is revolved about the horizontal line $y = -1$.

| Answers | Scoring Rubric |
|---|---|
| (a) .065<br><br>(b) .410<br><br>(c) $1.451\pi$ *or* 4.559 | (a)<br>3 : { 1 : limits; 1 : integrand; 1 : answer }<br>(b)<br>3 : { 1 : limits; 1 : integrand; 1 : answer }<br>(c)<br>3 : { 2 : integrand; 1 : limits, constant, and answer } |

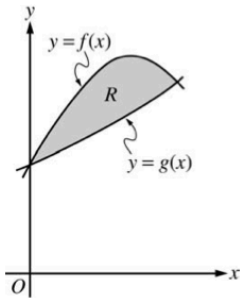
## 2005 (Calculator)

Let $f$ and $g$ be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let $R$ be the shaded region in the first quadrant enclosed by the graphs of $f$ and $g$ as shown in the figure above.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

(c) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $x$-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.




| Answers | Scoring Rubric |
|---|---|
| (a) .429<br><br>(b) 1.358$\pi$ *or* 4.267<br><br>(c) .025$\pi$ *or* .078 | 1 : correct limits in an integral in (a), (b), or (c)<br><br>(a)<br>2 : { 1 : integrand; 1 : answer }<br><br>(b)<br>3 : { 2 : integrand; ⟨−1⟩ each error; Note: 0/2 if integral not of form $c\int_a^b \left(R^2(x) - r^2(x)\right) dx$; 1 : answer }<br><br>(c)<br>3 : { 2 : integrand; 1 : answer } |

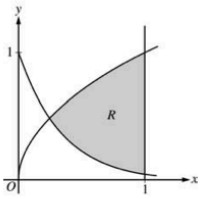
## 2003 (Calculator)

Let $R$ be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the horizontal line $y = 1$.

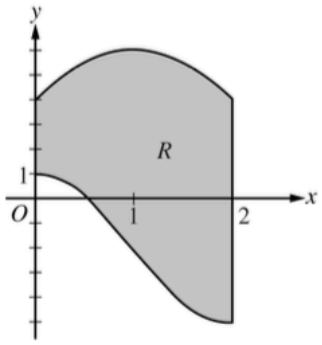
(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a rectangle whose height is 5 times the length of its base in region $R$. Find the volume of this solid.




| Answers | Scoring Rubric |
|---|---|
| (a) .443<br><br>(b) .453$\pi$ *or* 1.424<br><br>(c) 1.554 | 1 : correct limits in an integral in (a), (b), or (c)<br><br>(a)<br>2 : { 1 : integrand; 1 : answer }<br><br>(b)<br>3 : { 2 : integrand; < −1 > reversal; < −1 > error with constant; < −1 > omits 1 in one radius; < −2 > other errors; 1 : answer }<br><br>(c)<br>3 : { 2 : integrand; < −1 > incorrect but has $\sqrt{x} - e^{-3x}$ as a factor; 1 : answer } |

## 2019

. Let $R$ be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the $y$-axis, and the vertical line $x = 2$, as shown in the figure above.




(a) Find the area of $R$.

(b) Region $R$ is the base of a solid. For the solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 6$.

| Answers | Scoring Rubric |
|---|---|
| (a) $\frac{44}{3}$ | (a) $4:\begin{cases}1:\text{integrand}\\1:\text{antiderivative of } 3\cos\left(\frac{\pi}{2}x\right)\\1:\text{antiderivative of remaining terms}\\1:\text{answer}\end{cases}$ |
| (b) $\int_0^2 \frac{1}{x+3}\,dx$ *We will learn how to find solve this to get the answer in Unit 7. | (b) $2:\begin{cases}1:\text{integral}\\1:\text{answer}\end{cases}$ |
| (c) $\pi\int_0^2 \left((6-g(x))^2-(6-h(x))^2\right)dx$ | (c) $3:\begin{cases}1:\text{limits and constant}\\1:\text{form of integrand}\\1:\text{integrand}\end{cases}$ |