Chapter 14 Review Name key

More Proofs Practice

Prove each of the following.

1) $1+\sin 2\theta = (\sin\theta+\cos\theta)^2$

$(\sin\theta+\cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 + 2\sin\theta\cos\theta = 1+\sin 2\theta$ ✓

2) $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\tan^2\theta}{\sec^2\theta} = \dfrac{1}{\sec^2\theta} - \dfrac{\tan^2\theta}{\sec^2\theta} = \cos^2\theta - \dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta$

$= \cos^2\theta - \sin^2\theta = \cos 2\theta$ ✓

3) $\sec^2\theta = \dfrac{2}{1+\cos 2\theta}$

$\dfrac{2}{1+\cos 2\theta} = \dfrac{2}{1+2\cos^2\theta - 1} = \dfrac{2}{2\cos^2\theta} = \dfrac{1}{\cos^2\theta} = \sec^2\theta$ ✓

4) $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta} = -\sec 2\theta$

$\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta} = \dfrac{1}{-(\cos^2\theta-\sin^2\theta)} = \dfrac{1}{-\cos 2\theta} = -\sec 2\theta$ ✓

5) $\dfrac{(\sin\theta+\cos\theta)^2}{\sin 2\theta} = \csc 2\theta + 1$

$\dfrac{(\sin\theta+\cos\theta)^2}{\sin 2\theta} = \dfrac{\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta}{2\sin\theta\cos\theta} = \dfrac{1+\sin 2\theta}{\sin 2\theta}$

$= \dfrac{1}{\sin 2\theta} + \dfrac{\sin 2\theta}{\sin 2\theta} = \csc 2\theta + 1$ ✓

Use Sum/Difference Identities to find the exact value:

6. $\sin(195°) = \sin(150° + 45°)$

$= \sin 150° \cos 45° + \cos 150° \sin 45°$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

9. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2=2$

$\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta$

$=\sin^2\theta+\cos^2\theta+\sin^2\theta+\cos^2\theta$

$=1+1=2$ ✓

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10. $(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)=\sec\theta+\csc\theta$

$\sin\theta\tan\theta+\sin\theta\cot\theta+\cos\theta\tan\theta+\cos\theta\cot\theta$

$\sin\theta\cdot\frac{\sin\theta}{\cos\theta}+\sin\theta\cdot\frac{\cos\theta}{\sin\theta}+\cos\theta\frac{\sin\theta}{\cos\theta}+\cos\theta\frac{\cos\theta}{\sin\theta}$

$=\frac{\sin^2\theta}{\cos\theta}+\cos\theta+\sin\theta+\frac{\cos^2\theta}{\sin\theta}$

$=\frac{1-\cos^2\theta}{\cos\theta}+\cos\theta+\sin\theta+\frac{1-\sin^2\theta}{\sin\theta}$

$=\frac{1}{\cos\theta}-\frac{\cos^2\theta}{\cos\theta}+\cos\theta+\sin\theta+\frac{1}{\sin\theta}-\frac{\sin^2\theta}{\sin\theta}$

$=\sec\theta-\cos\theta+\cos\theta+\sin\theta+\csc\theta-\sin\theta$

$=\sec\theta+\csc\theta$ ✓

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11. $\frac{\tan\theta-1}{\tan\theta+1}=\frac{1-\cot\theta}{1+\cot\theta}$

$\frac{\tan\theta-1}{\tan\theta+1}=\left(\frac{\frac{1}{\cot\theta}-1}{\frac{1}{\cot\theta}+1}\right)\left(\frac{\cot\theta}{\cot\theta}\right)$

$=\frac{1-\cot\theta}{1+\cot\theta}$ ✓