Introduction to Counting

Example #1: **(Tree Diagram)**

You are at a carnival. One of the carnival games asks you to pick a door and then pick a curtain behind the door. There are 3 doors and 4 curtains behind each door. Create a tree diagram that represents this situation.

You try: **(Tree Diagram)**

A choice of pizza or spaghetti; a choice of milk or juice to drink; a choice of pudding or an apple for dessert

The **Fundamental Counting Principle** is a way to figure out the total number of ways different events can occur. The possible outcomes is the product of the decisions.

- When there are **m** ways to do one thing, and **n** ways to do another, then there are **m×n** ways of doing **both**.

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Example #2:

Yogurt Parfait (Choose 1 of each)		
Flavor	**Fruit**	**Nuts**
Plain	Peaches	Almonds
Vanilla	Strawberries	Peanuts
	Bananas	Walnuts
	Raspberries	
	Blueberries	

To make a yogurt parfait, you choose one flavor of yogurt, one fruit topping, and one nut topping. How many parfait choices are there?

Example #3:
There are 10 questions on a True/False test. How many different outcomes are there of student responses?

Factorial -- the product of the natural numbers less than or equal to the number.

- Let *n* be the number of items: $n \bullet (n-1) \bullet (n-2) \bullet (n-3) \ldots (1)$

Example: Find 6!

Permutations -- All possible arrangements of a collection of items where the order is important. ORDER MATTERS!

Example #4:
Suppose that I wanted to call on three of you to answer 3 different questions. I only want 3 out of 30 students. How many possibilities for students can I have?

Example #5:

A coach must choose how to line up his five starters from a team of 12 players. How many different ways can the coach choose the starters' arrangement?

Example #6:

STANDARD DECK = 52 cards

SUITS: *13 hearts, 13 diamonds, 13 spades, 13 clubs*

- *Hearts and diamonds are red*
- *Spades and clubs are black*
- *There are 10 number cards (including the Ace) in each suit.*
- *There are 3 face cards (Jack, Queen, and King) in each suit.*

a) How many possibilities are there of drawing 2 black cards and 1 red card when drawing 3 cards without replacement?

b) How many possibilities of drawing three kings, 1 queen, and a "6" when drawing 5 cards without replacement?

Permutations

NUMBERS	ALGEBRA
The number of permutations of 7 items taken 3 at a time is $${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}.$$	The number of permutations of n items taken r at a time is $${}_nP_r = \frac{n!}{(n-r)!}.$$

A **combination** is a grouping of items in which order does NOT matter. There are generally fewer ways to select items when order does not matter.

- Think of as "choosing" not "arranging"

6 permutations → {ABC, ACB, BAC, BCA, CAB, CBA}
1 combination → {ABC}

Example #7: The track team has 10 qualified runners for the 2-mile race. Three runners will be selected to run in the first heat. How many ways can the runners be selected?
(Notice how I did not ask how I could order them for the race)

Example #8: A team of 17 softball players needs to choose three players to refill the water cooler. How many ways could these players be selected?

Combinations

NUMBERS	ALGEBRA
The number of combinations of 7 items taken 3 at a time is $_7C_3 = \frac{7!}{3!(7-3)!}$.	The number of combinations of n items taken r at a time is $_nC_r = \frac{n!}{r!(n-r)!}$.

Find $_{10}C_4$ and $_{12}C_7$. How do these compare to $_{10}P_4$ and $_{12}P_7$?