The Natural Base "e"

The number ***e*** is a famous irrational number, and is one of the most important numbers in mathematics. The first few digits are

2.71828182845904523536028747135 27…

It is often called **Euler's number** after Leonhard Euler.

- ***e*** is the base of the natural logarithms

Natural Logarithm

- A logarithm with a base of *e*, but instead of writing $\log_e$ we write as ln.

Example #1: Convert between exponential and logarithmic form.

$e^x = 11$ ln(x)=8

Example #2: **Solve each.**

$e^x = 7$ $e^{4x} = 19$

Example #3: **Solve each.**

lnx = 3 4lny = 12

You try: Convert to logarithmic form.

$e^x = 4$

Example #2: **Simplify each of the following expressions.**

$\boldsymbol{e^{ln2}}$ **3lnx**

$\boldsymbol{e^{5lnx}}$ **ln($\boldsymbol{e^4}$)**

ln($\boldsymbol{e}$) **ln(x) + ln(3) - ln(t)**

You try: Simplify each.

A) $\boldsymbol{e^{3lnx}}$ B) **4lnx + ln5**

Discuss: Can log7 + ln6 be simplified any further? Explain.

Example #4:

Solve $ln5 + lnx = 1$

Example #5:

Solve $e^{3lnx} = 8$

You try: Solve $2lnx - 2 = 0$

Continuously Compounded Interest

- Compounded interest is when interest (a fee) is added to a deposit or loan, so that, from that moment on, the interest that has been added also earns interest.
- $A = Pe^{rt}$, where A is the total amount, P is the principal, r is the annual interest rate, and t is the time in years.

Example #4:
What is the total amount for an investment of $500 invested at 5.25% for 40 years and compounded continuously?

How long will it take for the investment to reach $2500?

You try:
What is the total amount for an investment of $100 invested at 3.5% for 8 years and compounded continuously?

Partner Practice:

1) The grandparents of a newborn child decide to establish a college fund for their new granddaughter. They invest $10,000 in a fund that pays 4.5% interest compounded continuously. Write a model for the value of the investment over time.

2) In 2013, the average annual public in-state tuition was $8900, which was 2.3% above the 2012 average. If this trend continues, write a model that represents the amount of money needed to pay one year's tuition in years, *t*, after 2013.

3) In the year 2031, the granddaughter will be 18 and entering college. How many years will it take for the grandparents to save enough money for her first year of tuition?