## Logarithms

A logarithm is the inverse operation that undoes raising a base to an exponent equation


Read $\log _{\mathrm{b}} a=x$, as "the $\log$ base $b$ of $a$ is $x$."
$>$ Notice that the log is equal the exponent of the exponential form.

Example \#1: Rewrite as a logarithm.

$$
6^{3}=216
$$

$$
4^{6}=4096
$$

You try: $10^{5}=100000$

Example \#2: Rewrite as an exponential.
$\log _{7} x=2$

$$
\log _{x} 27=3
$$

You try: $\log _{10} x=7$

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Example \#3: Simplify each expression.
$\log _{4} 64$
$\log _{2} 64$
$\log _{3} 81$

A logarithm with base 10 is called a common logarithm. If no base is written for a logarithm, the base is assumed to be 10.

Example \#3:
$\log 100,000$

Example \#4: Solve $\boldsymbol{\operatorname { l o g } \boldsymbol { x }}=\mathbf{3}$.

You try: Solve $\log \boldsymbol{x}=\mathbf{- 1}$.
$>$ Does $\log _{0} 5$ exist? Explain.
$>$ Does $\log (-3)$ exist?
$>$ What is $\log 1$ ?

## Properties of Logarithms

| Property | Definition | Example |
| :--- | :---: | :---: |
| Product | $\log _{b} m n=\log _{b} m+\log _{b} n$ | $\log _{3} 9 x=\log _{3} 9+\log _{3} x$ |
| Quotient | $\log _{b} \frac{m}{n}=\log _{b} m-\log _{b} n$ | $\log _{\frac{1}{4}} \frac{4}{5}=\log _{\frac{1}{4}} 4-\log _{\frac{1}{4}} 5$ |
| Power | $\log _{b} m^{p}=p \cdot \log _{b} m$ | $\log _{2} 8^{x}=x \cdot \log _{2} 8$ |

Example \#1: Expand $\log (a b c)$

Example \#2: Expand $\log \left(x^{3}\right)$

Example \#3: Expand $\log \left(\frac{3 x}{4 m}\right)$

You try: Expand $\log (4 x y)$
Expand $\log \left(\frac{w}{2 x}\right)$

Example \#4: Rewrite as a single logarithm.

$$
\log 2+\log 3-\log 8
$$

Example \#5:

$$
3 \log x-\log (x)
$$

You try: Rewrite as a single logarithm: 4logw $+\log 5$

## Exponent Properties:

$$
\begin{array}{ll}
\log _{b} b^{x}=x & b^{\log _{b} x}=x \\
\log 10^{x}=x & 10^{\log x}=x
\end{array}
$$

Example \#6: Simplify $\log _{3} 3^{4}$

Simplify: $3 \log _{5} 5$

Example \#7: Solve for $\mathbf{x}: \log _{7}\left(7^{3 x}\right)=9$

You try: Simplify $\log _{2} 2^{9}$
Simplify: $\log 3+\log x-\log 7$

Lesson 22-2 and 23-2

## Relating to Inverses:

Solve for the inverse of $f(x)=2^{x}$

Solve for the inverse of $f(x)=4 \cdot 2^{x}+2$

