

Infinite Geometric Series

Practice:

For each Geometric series, find the partial sums for the first 3 terms, first 4 terms, and first 6 terms.

$$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \quad R_n = \frac{1}{32} + \frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \dots$$

When $|r| < 1$, the partial sum approaches a fixed number and the series is said to **converge**.

When $|r| \geq 1$, the partial sum does not approach a fixed number and the series is said to **diverge**.

Example #1:

Determine whether each geometric series converges or diverges.

A. $10 + 1 + 0.1 + 0.01 + \dots$

B. $4 + 12 + 36 + 108 + \dots$

C.

$$\sum_{k=1}^{\infty} 25 \left(\frac{1}{5}\right)^{k-1}$$

If an infinite series converges, we can find the sum.

Sum of an Infinite Geometric Series

The sum of an infinite geometric series S with common ratio r and $|r| < 1$ is

$$S = \frac{a_1}{1 - r},$$

where a_1 is the first term.

Example #2:

Find the sum, if it exists: $1 - 0.2 + 0.04 - 0.008 + \dots$

You try: **Find the sum, if it exists.**

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-1}$$

$$\sum_{k=1}^{\infty} \frac{3}{4} (7)^{k-1}$$

$$\sum_{k=1}^{\infty} 3 \left(\frac{2}{5}\right)^{k-1}$$