Introduction to Series and Summation Notation

A **series** is the indicated sum of the terms of a sequence.

Sequence	1, 2, 3, 4	2, 4, 6, 8, ...	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
Series	$1 + 2 + 3 + 4$	$2 + 4 + 6 + 8 + \cdots$	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

A **partial sum**, indicated by S_n, is the sum of a specified number of terms of a sequence.

Example #1:
Find S_5 for the even numbers sequence: 2, 4, 6, 8, ...

A series can also be represented by using **summation notation**, which uses the Greek letter Σ (capital *sigma*) to denote the sum of a sequence defined by a rule.

$$\sum_{k=1}^{5} 2k$$

5 ← **Last value of k**

Σ2k ← **Explicit formula for sequence**

$k=1$ ← **First value of k**

Write the series in summation notation:
   1) 3 +6 +9 +12

   2) 4 + 9 +14 +19 +24 +29

3) 8 +16+ 32 + 64 + 128

4) -1 + 2 - 4 + 8

Example #2: **Expand the series and evaluate.**

$$\sum_{k=1}^{6}\left(k^2 - 10\right)$$

Example #3: **Expand and evaluate.**

$$\sum_{k=1}^{5}-5(2)^{k-1}$$

You try:

$$\sum_{k=1}^{7}3$$

In a *constant series,* each term has the same value.

Arithmetic and Geometric Series

Arithmetic Series

- The sum of an arithmetic sequence.

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

- n is the number of terms, a_1 is the 1st term, and a_n is the last term.

Example #1:
Find the sum of 2 + 4 + 6 + 8 + 10

Example #2:
The side section of an auditorium has 12 seats in the first row and 3 additional seats in each subsequent row. How many seats are in the 10th row? How many seats in total are in the first 10 rows?

Example #3:

$$\sum_{k=1}^{15} (5 + 2k)$$

Geometric Series – The sum of a geometric sequence.

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right), \; r \neq 1$$

- n is the number of terms, a_1 is the 1st term, and r is the common ratio.

Example#4:
Find S_8 for **1 + 2 + 4 + 8 + 16 + ...**

Example #5:

$$\sum_{k=1}^{6}\left(\frac{1}{2}\right)^{k-1}$$

You try: **Find S_5 for the sequence: 32, 8, 2, .5, ...**

Example #6: **A 6-year lease states that the annual rent for an office space is $84,000 the first year and will increase by 8% each additional year of the lease. What will the total rent expense be over the 6-year lease?**