Arithmetic and Geometric Sequences

A **sequence** is a set of numbers in a particular order or pattern.

- Example: Fibonacci Sequence: 0, 1, 1, 2, 3, 5, 8, 13, …

A **Term** is each number in a sequence.

- The first term is denoted by a_1 (Read as: *a* subscript 1)
- The second term is denoted by a_2, third term is denoted by a_3, and so on.
- All term subscripts are made of natural numbers

You try: What is a_5 of the Fibonacci Sequence?

A **Finite Sequence** contains a limited number of terms.

- Example: {-2, 0, 2, 4, 6}

An **Infinite Sequence** continues without end.

- Example: Fibonacci Sequence

Example #1:
{2, 4, 8, 16, 32}

Example #2:
2, 4, 8, 16, 32, …

Arithmetic Sequence

- Each term is determined by adding a constant value to the previous term.
- The added constant value is called the common difference.
  - Example: {3, 6, 9, 12, 15}

Geometric Sequence

- Each term is determined by multiplying a nonzero constant by the previous term.
- The multiplied nonzero constant is called the common ratio.

Determine whether each sequence is arithmetic or geometric.

(1) 2, 6, 18, 54, …

(2) 5, -6, -17, -28, …

(3) -4, 12, 28, 44, …

Types of Formulas for Sequences

Explicit Formula

- Defines the *n*th term of a sequence as a function of *n.*

Arithmetic Sequence – Linear

- Formula: $\boldsymbol{a_n = a_1 + (n - 1)d}$

Geometric Sequence – Exponential

- Formula: $a_n = a_1 \left(r^{n-1}\right)$

Example #3: Write the explicit formula for each of the 3 previous sequences.

(1) 2, 6, 18, 54, …

(2) 5, -6, -17, -28, …

(3) -4, 12, 28, 44, …

Recursive Formula

- A rule in which one or more previous terms are used to generate the next term.
    - Example: Fibonacci Sequence

Example #4: **Find the first 5 terms in the sequence.**

$a_n = 2a_{n-1} + a_{n-2}$ where $a_1 = 2$ and $a_2 = 4$

You try: **Find the first 4 terms of the sequence:**

$a_n = -a_{n-1} + 5$ where $a_1 = 3$

Example #5: **Find the indicated terms in this arithmetic sequence:** 11, _, _, _, -17

Example #6: **Find the 10th term of the geometric sequence that has $a_5 = 96$ and $a_7 = 384$.**

You try: **Geometric Sequence:** 4, ____, 36, …

Arithmetic Sequence: $\frac{1}{2}$, ____ , _____ , 2