

Spring Final Review

1. (a) $\log_4\left(\frac{8}{2}\right) = 1$

(b) $\log 15$

(c) $\log_4\left(\frac{24}{2}\right) = \log_4 12$

9. $2 + 2^2 + 2^3 + 2^4$

$= \boxed{30}$

2. $\frac{x+5+b}{5} = \boxed{\frac{x+11}{5}}$

3. $4^x = \frac{1}{16} \quad \boxed{x = -2}$

4. $\log_3 \frac{x^2}{4} = 2$

$3^2 = \frac{x^2}{4}$

$x^2 = 36 \quad \boxed{x = 6}$

10. $a_{30} = 40 + (29)(-10)$

$= -250$

$S_{30} = 30 \left(\frac{-250 + 40}{2} \right)$

$= \boxed{-3150}$

11. $r^2 = \frac{384}{96} = 4 \quad r = \pm 2$

$a_{10} = \pm 3072$

5. $5\pi/6$

6. (a) Reflect across x-axis
Horizontal compression
by $1/3$

Translation up 5

$x = 0$

(b) vertical stretch by 4
Reflect across y-axis
Translation down 3

$y = -3$

12. $S = 50(.8)^{n-1}$
4.29 million

13. $\frac{3x^2 - 1 - (x+2)(x+3)}{(x-6)(x+3)}$

$\frac{x^2 + 5x + 6}{3x^2 - 1 - (x^2 + 2x + 3x + 6)}$

$3x^2 - x^2 - 5x - 1 - 6$

$= \boxed{\frac{2x^2 - 5x - 7}{x^2 - 3x - 18}}$

7. $r = 1/3$

8. $S = \frac{6}{1 - 1/10} = \frac{6}{9/10} = 6 \cdot \frac{10}{9} = \frac{6 \cdot 67}{9} = \boxed{\frac{20}{3}}$

$$14. \frac{2 \cdot 4x^2(x+3)}{2x^2(x+8)} \cdot \frac{(x-8)(x+3)}{x+3} = 4(x+3)$$

$$15. \frac{1}{x-1} + \frac{4}{x+1} = \frac{7}{x^2-1}$$

$$\frac{x+1+4x-4}{x^2-1} = \frac{7}{x^2-1}$$

$$5x-3=7$$

$$\boxed{x=2}$$

$$16. x^2+12x+36=4x+21$$

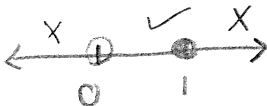
$$x^2+8x+15=0$$

$$(x+5)(x+3)=0$$

$$\boxed{x=-5, -3}$$

$$17. \frac{2x+1}{x} \geq 3 \quad x \neq 0$$

$$2x+1=3x$$

$$x=1$$


$$\boxed{[0, 1]}$$

$$18. f(x) = \frac{x-2}{x^2-2x-3} =$$

- (a) $x=2$
- (b) $x=3, x=-1, y=0$
- (c) As $x \rightarrow \pm\infty, y \rightarrow 0$

$$19. \left(\frac{1}{4}\right)^x = 8^{x-1}$$

$$2^{-2x} = 2^{3(x-1)}$$

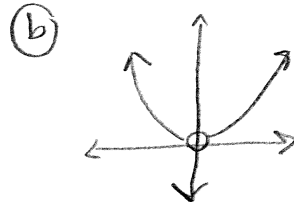
$$-2x = 3x-3$$

$$3 = 5x$$

$$\boxed{x = 3/5}$$

$$20. f(x) = x^5/x^3 = x^2$$

(a) Hole: $x=0$



21. Reflect across y-axis
Right 2, up 3

$$22. x+6 = 8(x-1)$$

$$x+6 = 8x-8$$

$$14 = 7x \quad \boxed{x=2}$$

$$23. (a) x+7, x-7$$

$$(b) t(x) = \frac{3}{x+7} + \frac{3}{x-7}$$

24. (a) $x = -4, y = -3$
 (b) As $x \rightarrow \pm\infty, y \rightarrow -3$.
 (c) D: $x \neq -4$ R: $y \neq -3$
 (d) increasing: never
 dec: $(-\infty, -4) \cup (-4, \infty)$

25. $x^2 - 16x + 64 + y^2 + 10y + 25 = -53 + 64 + 25$
 $(x-8)^2 + (y+5)^2 = 36$
 circle

26. (a) parabola
 (b) ellipse
 (c) hyperbola
 (d) circle

27. $\frac{(x+9)^2}{81} + \frac{(y+4)^2}{9} = 1$
 ellipse
 $a = 9$ $b = 3$
 Major: 18 Minor: 6
 $c = \sqrt{81 - 9} = \sqrt{72} = 6\sqrt{2}$

28. $\tan(-195^\circ) = -\tan 195^\circ$
 $\tan(150+45) = \frac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45}$
 $= \frac{-\frac{\sqrt{3}}{3} + 1}{1 + \frac{\sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$
 $\tan(-195^\circ) = \frac{\sqrt{3} - 3}{3 + \sqrt{3}}$

$$29. \frac{y^2}{25} - \frac{(x+2)^2}{49} = 1$$

$$a=5 \quad b=7$$

$$\text{Trans: } 10$$

$$\text{Conj: } 14$$

$$30. F: (-4, 0) \quad x=2$$

$$p=-3$$

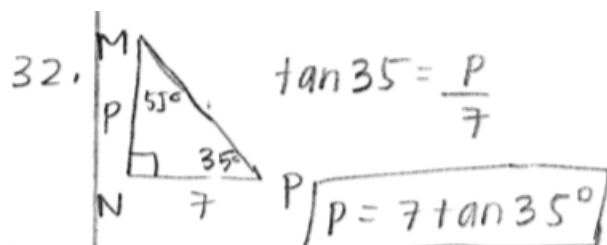
$$v: (-1, 0)$$

$$x+1 = \frac{-1}{12} y^2$$

$$31. \frac{9+x}{2} = 6 \quad \frac{4+y}{2} = 8$$

$$9+x=12 \quad x=3 \quad y=12$$

$$A: (3, 12)$$



$$33. \frac{\cos \theta (2 \sin \theta \cos \theta)}{1 + 2 \cos^2 \theta - 1}$$

$$= \frac{2 \cos^2 \theta \sin \theta}{2 \cos^2 \theta} = \sin \theta \checkmark$$

$$37. \csc 540^\circ = \text{undefined}$$

$$38. f(x) = -\frac{1}{2} \cos 2\left(\theta + \frac{\pi}{2}\right) - 3$$

$$A = \frac{1}{2} \quad p = \pi \quad b = 2$$

$$\text{phase shift: } -\pi/2$$

$$\text{Mid: } y = -3 \quad D: \mathbb{R}$$

$$R: [-3.5, -2.5]$$



$$\sin \theta = 115/160$$

$$\theta = 46^\circ$$

41.

$$\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x \cdot (1 + \cos x)}{1 - \cos x \cdot (1 + \cos x)} = \frac{\csc x (1 + \cos x)}{1 - \cos^2 x} = \frac{1 + \cos x}{\sin x (\sin^2 x)} = \frac{1 + \cos x}{\sin^3 x} \checkmark$$

$$\frac{\cos x + 1}{\sin^3 x} = \frac{\cos x + 1}{\sin x (1 - \cos^2 x)} = \frac{\cancel{\cos x + 1}}{\sin x (1 + \cos x)(1 - \cos x)} = \frac{\csc x}{1 - \cos x} \checkmark$$