<u>Double Angle Identities</u>

Instead of the angle being just "θ", the angle will be "2θ"

Double Angle:

$$\sin 2\theta = \sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$$
$$= \boxed{2\sin\theta\cos\theta}$$

We can use the Sum/Difference Identities to derive the Double Angle Identities.

$$\cos 2\theta = \cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta = \boxed{\cos^2\theta - \sin^2\theta}$$

$$1-\sin^2\theta-\sin^2\theta = \boxed{1-2\sin^2\theta} \qquad \cos^2\theta-(1-\cos^2\theta) = \boxed{2\cos^2\theta-1}$$

Double-Angle Identities		
$\sin 2\theta = 2\sin\theta\cos\theta$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$ $\cos 2\theta = 1 - 2\sin^2\theta$	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Recall from the Pythagorean Identities that $cos^2\theta + sin^2\theta = 1$, which means that $cos^2\theta = 1 - sin^2\theta$ and $sin^2\theta = 1 - cos^2\theta$. This is why $cos2\theta$ has so many variations.

NOTE: It is very important to remember that $cos2\theta \neq 2cos\theta$. The interior coefficient cannot be pulled to the front! In terms of transformations, this should make sense since $2cos\theta$ indicates that the amplitude has changed to be 2, where $cos2\theta$ indicates a period/frequency change.

<u>Example #1:</u>

Prove: $\sin 2\theta = 2\tan\theta - 2\tan\theta\sin^2\theta$

$$2\tan\theta - 2\tan\theta\sin^2\theta = 2\tan\theta(1-\sin^2\theta)$$
$$= 2\cdot\frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cos^{\cancel{2}}\theta$$
$$= 2\sin\theta\cos\theta$$
$$= \sin 2\theta \checkmark$$

1. I factored out a common term.

2. I made a substitution using a Pythagorean Identity.

3. When the remaining expression is simplified, it matches the desired result!

Since $cos2\theta$ has so many variations, it will be important in proofs to be strategic about which form we select. Since all 3 forms are equivalent, they will all eventually lead you to a complete proof, but there is always a fastest option. I will discuss this in the example proof below.

Example #2:

$\cos 2\theta = (2 - \sec^2\theta)(1 - \sin^2\theta)$

$(2-\sec^2\theta)(1-\sin^2\theta) = \cos^2\theta(2-\sec^2\theta)$

$= 2\cos^2\theta - \cos^2\theta\sec^2\theta$

$= 2\cos^2\theta - 1$

$= \cos 2\theta$ ✓

1. I made a substitution using $cos^2\theta = 1 - sin^2\theta$

2. I distributed.

3. $cos^2\theta sec^2\theta = cos^2\theta \cdot \frac{1}{cos^2\theta} = 1$

4. The remaining expression simplifies to match one of the forms of $cos2\theta$

Section 14.5

You try: **$\sin 2x = 2\cot(x)\sin^2(x)$**

Sample Proof:

$2\cot(x)\sin^2(x) = 2\frac{cosx}{sinx} \cdot sin^2x = 2cosxsinx = sin2x$