Section 14.3

**All identities need to be memorized!

Fundamental Trigonometric Identities

Reciprocal Identities:

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \dfrac{1}{\csc\theta} = \sin\theta$

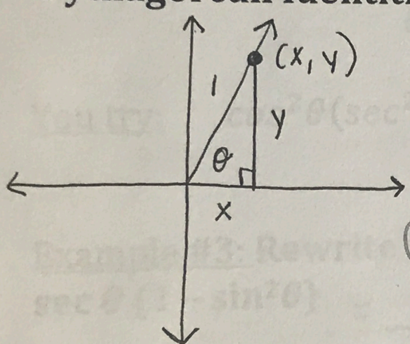
$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \dfrac{1}{\sec\theta} = \cos\theta$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\qquad \dfrac{1}{\cot\theta} = \tan\theta$

Tangent and Cotangent ratios:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities:



$$x^2 + y^2 = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

1 $\boxed{\cos^2\theta + \sin^2\theta = 1}$ $\qquad 1 - \sin^2\theta = \cos^2\theta$

Recall that $x = \cos\theta$ and $y = \sin\theta$ from the Unit Circle lesson.

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \qquad \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta} \qquad \boxed{\cot^2\theta + 1 = \csc^2\theta}$$

Note: Instead of writing $(cos\theta)^2$, we write $cos^2\theta$. Trig exponents are moved to be between the name of the ratio and the angle to minimize the use of parentheses.

This lesson derives the Pythagorean Identities,
but only the boxed identities are what need to be memorized for future use.
The 3 Pythagorean Identities may also be rearranged to find an equivalent form.
For instance, if $cos^2\theta + sin^2\theta = 1$, then $cos^2\theta = 1 - sin^2\theta$ and $sin^2\theta = 1 - cos^2\theta$.

Negative Angle Identities:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

We will be doing proofs using these new identities. When constructing a proof, we need to show that one side is equivalent to the other side. We do not use the two-column proof structure from geometry. Instead, we just show how to manipulate one side in order for it to become the other side. I recommend starting with the side that seems more complicated. The following examples demonstrate one example proof, but there are other correct intermediate steps that could have happened.

Section 14.3

Example #1: **Prove the trig identity.**

$$\tan\theta = \frac{\sec\theta}{\csc\theta}$$

$$\frac{\sec\theta}{\csc\theta} = \sec\theta \cdot \frac{1}{\csc\theta} = \frac{1}{\cos\theta} \cdot \sin\theta = \tan\theta \checkmark$$

Example #2: **Prove the trig identity.**

sin *θ* cot *θ* = cos *θ*

$$\sin\theta\cot\theta = \sin\theta \cdot \frac{\cos\theta}{\sin\theta} = \cos\theta \checkmark$$

You try: $cos^2\theta(sec^2\theta - 1) = sin^2\theta$

Sample answer:

$$cos^2\theta(sec^2\theta - 1) = cos^2\theta(tan^2\theta) = cos^2\theta\left(\frac{sin^2\theta}{cos^2\theta}\right) = sin^2\theta$$

Notice how in this proof, I used a rearranged version of a Pythagorean Identity.

If $1 + tan^2\theta = sec^2\theta$, then $sec^2\theta - 1 = tan^2\theta$

Example #3: **Rewrite in terms of cosine and then simplify.**

$$\sec\theta\,(1 - \sin^2\theta) = \frac{1}{\cos\theta} \cdot \cos^2\theta = \boxed{\cos\theta}$$

Example #4: **Rewrite in terms of cosine and then simplify.**

$$\frac{2(\csc^2\theta - \cot^2\theta)}{\sec\theta} = \frac{2 \cdot 1}{\sec\theta} = \boxed{2\cos\theta}$$

You try: **Simplify:** $csc\theta cos\theta tan\theta$

Sample answer:

$$csc\theta cos\theta tan\theta = \frac{1}{sin\theta} \cdot cos\theta \cdot \frac{sin\theta}{cos\theta} = 1$$