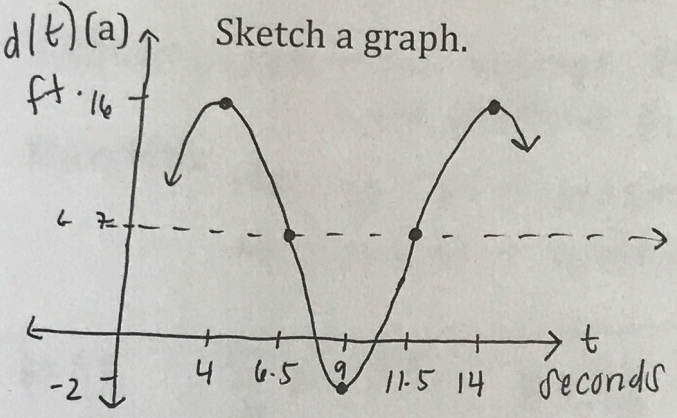
Word Problems with Sine and Cosine

Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, he noticed a dead fish caught on one of the paddles. As the wheel turned, the distance, d, that the fish was from the water's surface was a periodic function of time. When his stopwatch read 4 seconds, the fish was at its highest, 16 feet above the water's surface. It took another 10 seconds before the fish reached that height again. The diameter of the wheel was 18 feet.

(a) Sketch a graph.



$P = 10 \quad b = \frac{\pi}{5}$

Midline: $y = 7$

Since 10 seconds pass between two max values, the period is 10. We use this to find the frequency. Since the first max occurs at 4 seconds, we consider using a cosine function and "starting" it at 4 by making a phase shift. Since the wheel's diameter is 18, the lowest value must be -2. Halfway between those values must be the midline.

(b) Write an equation.

$$d(t) = 9\cos\left[\frac{\pi}{5}(t-4)\right] + 7$$

(c) How far above the surface was the ~~point~~ fish when Mark's stopwatch read 5seconds? 17 seconds?

$d(5) = 14.28$ ft. $\quad d(17) = 4.22$ ft.

Plug the function into the calculator to find each of these values.

(d) What is the first positive value of time at which the ~~point~~ fish was at the water's surface? At that time, was it going into or coming out of the water? Explain.

$9\cos\frac{\pi}{5}(t-4) + 7 = 0$

$t = .08$

coming out of the water because the graph is increasing.

Plug the function into the calculator. Find the zeros using 2nd TRACE.

The depth d in feet of the water in a bay at any time is given by $d(t) = \frac{3}{2}\sin(\frac{5\pi}{31}t) + 23$, where t is the time in hours.

Describe what each represents in context:

Amplitude: $3/2$ → amount that the water rises/lowers from starting amount

Period: $\frac{62}{5}$ → the amount time it takes for the water to rise once and lower once + return to start height.

Midline: $y = 23$ ft → the average depth of the water
↘ the starting depth of water

Max/Min:
Max: 24.5 ft → Maximum depth of water
Min: 21.5 ft → minimum depth of water.

$b = \frac{5\pi}{31}$ $\frac{5\pi}{31} p = 2\pi$ $p = \frac{62}{5} = 12.4$

Remember to use the frequency to solve for the period.