Transformations of Sine and Cosine

The equations for a wave function use our trig ratios.

Here is a link to a video showing the relationship between the wave and the unit circle:

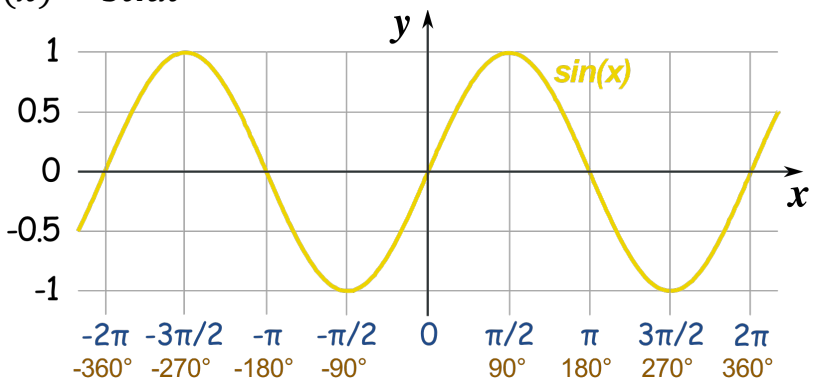
https://www.youtube.com/watch?v=Q55T6LeTvsA

To graph the function $\boldsymbol{f(x) = sinx}$, we imagine plugging in each angle value on the unit circle and then finding the sine of that value.

For instance, $f(0) = \sin(0) = 0$ and $f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

We could also consider any coterminal angles, allowing us to extend the graph to negative angle values as well as angle values larger than 2π.

Parent Function: $\boldsymbol{f(x) = sinx}$

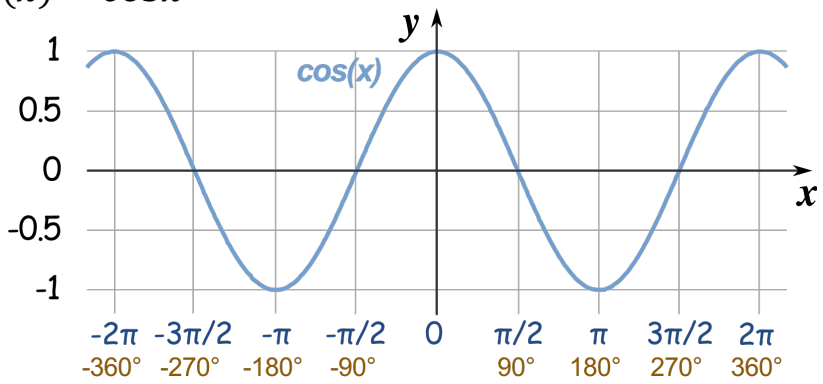


To graph the function $\boldsymbol{g(x) = cosx}$, we imagine plugging in each angle value on the unit circle and then finding the sine of that value.

For instance, $g(0) = \cos(0) = 1$ and $f\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

We could also consider any coterminal angles, allowing us to extend the graph to negative angle values as well as angle values larger than 2π.

Parent Function: $\boldsymbol{f(x) = cosx}$



Similarities between sine and cosine graphs:

Max Value: Min Value:

Midline: Amplitude:

Period: Frequency:

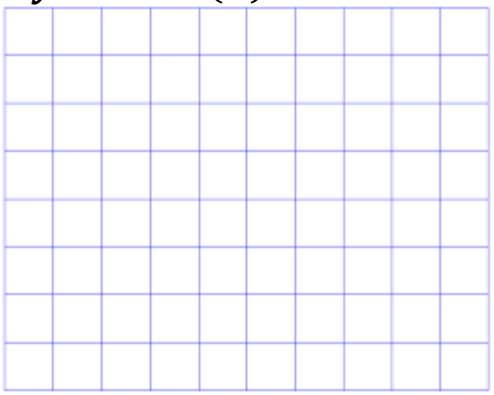
Differences between sine and cosine graphs:

When we graph transformations of sine and cosine, we will be transforming the 5 main points on the graph that correspond to a max, min, or midline value.
In the parent function, each of these key values are $\frac{\pi}{2}$ units apart. We will graph at least one period of each function. The examples will all be graphed in terms of radians.

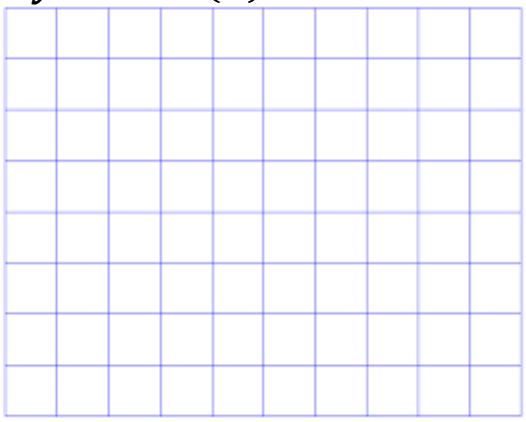
Transformations – **Amplitude change**

** The amplitude is the number multiplied in front of the trig function.

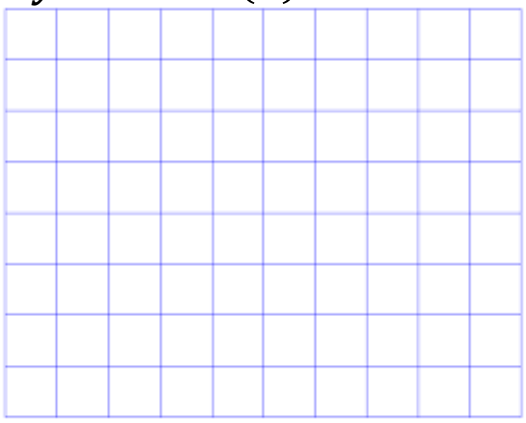
Graph $y = 2cos(\theta)$.



Graph $y = 5sin(\theta)$.



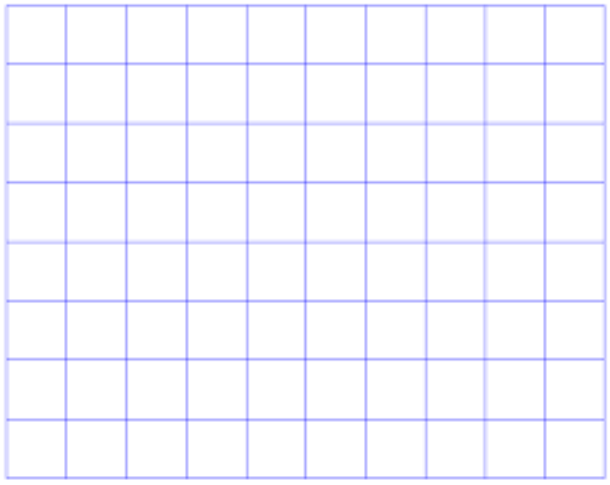
Graph $y = -2cos(\theta)$.



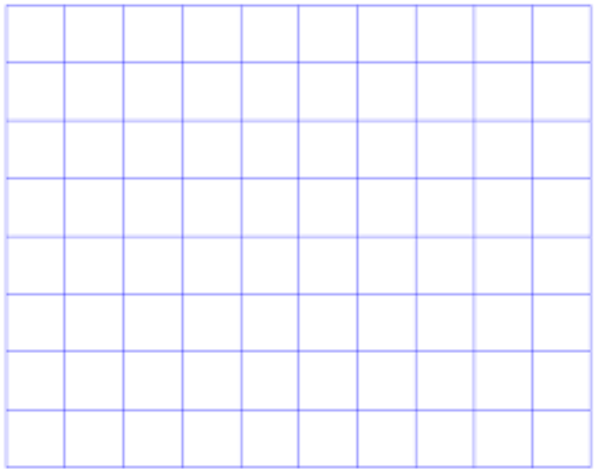
Remember that amplitude is a distance, so it must be a positive value! A negative in front of our function indicates a reflection across the x-axis, not a change in amplitude.

Vertical Shifts:

Example #1: **Graph $y = sinx + 3$**



Example #2: **Graph $y = 2cosx - 3$**

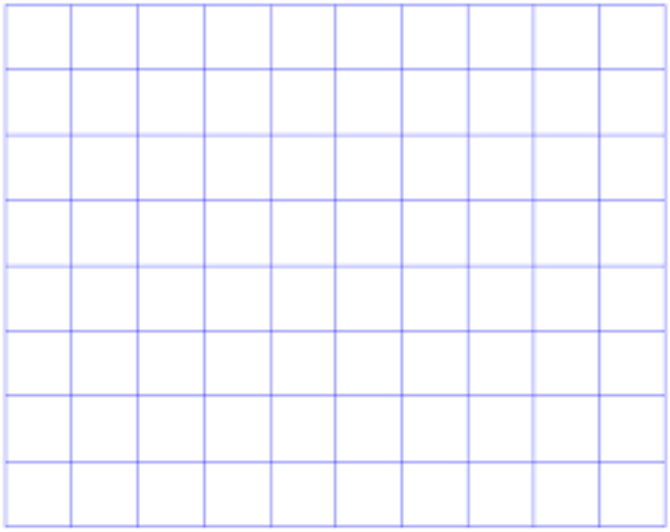


You try: Graph $k(x) = 3cosx + 2$

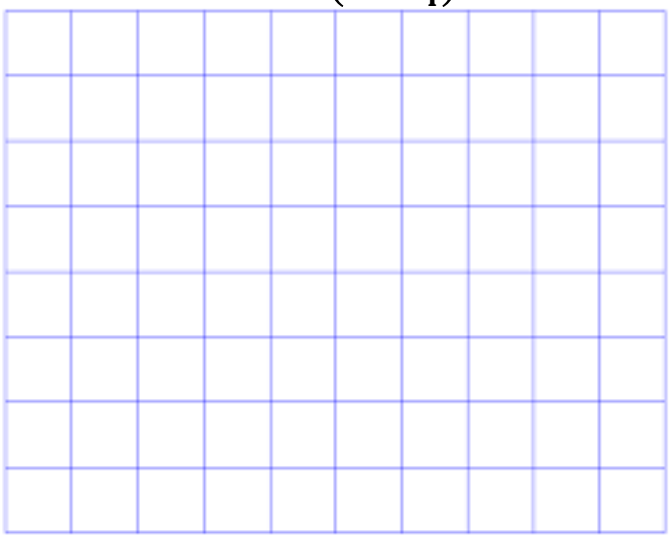
A **phase shift** is a horizontal shift of a wave function.

Example #3:

Using $f(x) = \sin x$ as a guide, graph $g(x) = sin\left(x - \frac{\pi}{4}\right)$.



Graph $g(x) = cos\left(x - \frac{\pi}{4}\right)$.

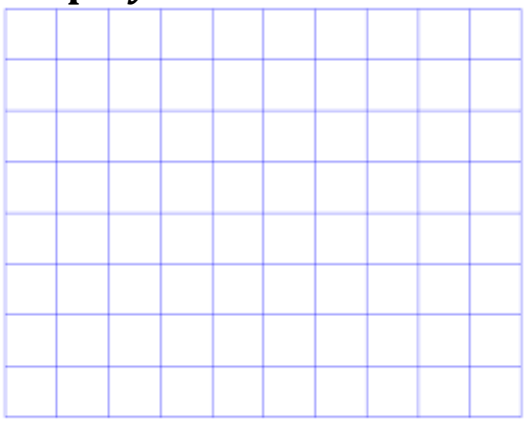


You try: **Graph $y = sin(x - \pi)$.**

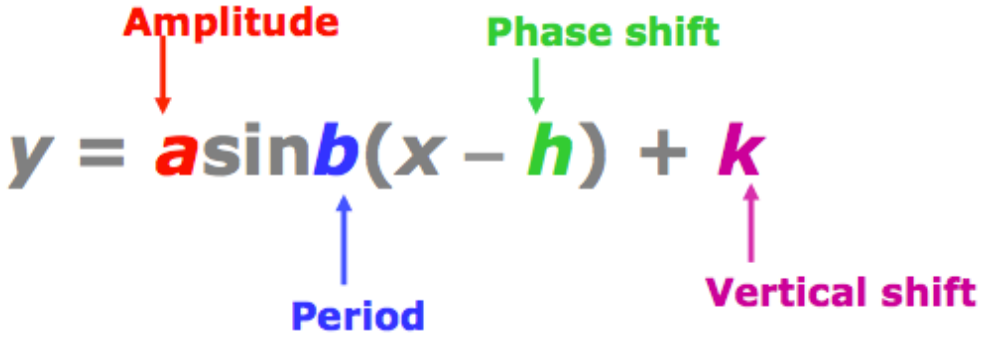
How can we write cosine as a phase shift of sine?

The last type of transformation is a period/frequency change.

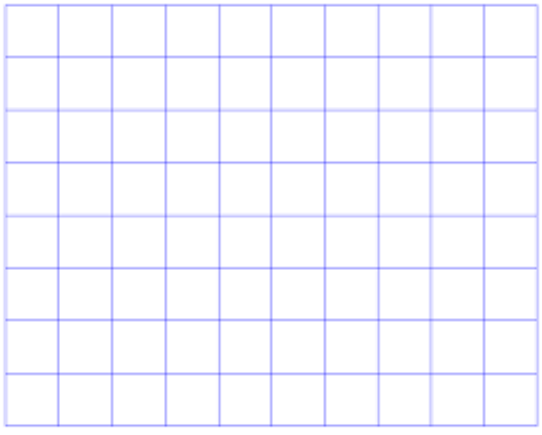
Graph $y = cos2x$.



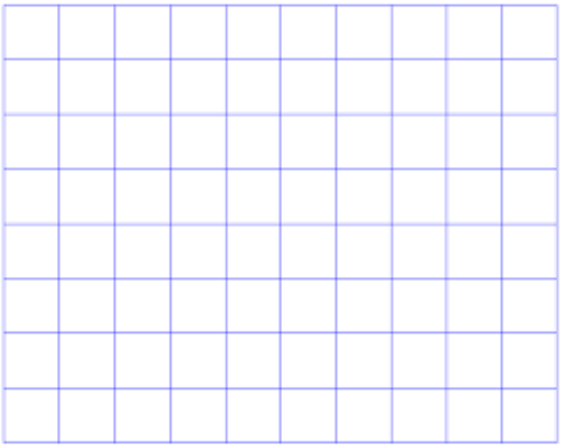
General form of transformations:



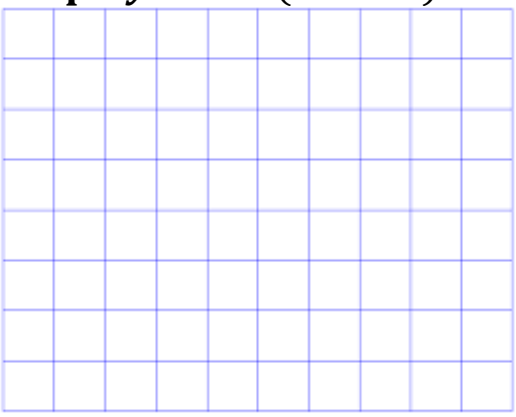
Graph $y = 2sin(\pi x) - 1$



Graph $y = 2sin2\left(x - \frac{\pi}{2}\right) - 1$



Graph $y = cos(3x - \pi) + 3$



You try: **Graph $y = cos\left(2x - \frac{\pi}{2}\right) - 2$**