For this lesson, you will want to have a filled-in Unit Circle to reference. The entire first page of the notes are non-calculator questions because they directly relate to the Unit Circle angles.

Inverse Trig

Used to "free" a variable from inside of a trig function.

Example: $\cos(x) = \frac{1}{2}$

$\cos^{-1}(\cos(x)) = \cos^{-1}(\frac{1}{2})$

$x = \cos^{-1}(\frac{1}{2})$

$\frac{\cos x}{\cos} = \frac{1/2}{\cos}$ ~~$x = \frac{1/2}{\cos}$~~

** You can't just divide by "cos", you need to use an inverse operation.

Function	Inverse Relation	Alternate Notation
$\sin\theta = a$	$\sin^{-1} a = \theta$	$\arcsin(a) = \theta$
$\cos\theta = a$	$\cos^{-1} a = \theta$	$\arccos(a) = \theta$
$\tan\theta = a$	$\tan^{-1} a = \theta$	$\arctan(a) = \theta$

"Find the inverse sine of ½"

Example #1: Find $sin^{-1}\left(\frac{1}{2}\right)$ using the unit circle.

(where does sine equal ½?)

$\frac{\pi}{6} + 2\pi n$
$\frac{5\pi}{6} + 2\pi n$
where n is an integer.

We look on the Unit Circle for any angles where the y-value would be ½, since this is a sine problem. In this case, there are two values in the circle, $\frac{\pi}{6}$ *and* $\frac{5\pi}{6}$

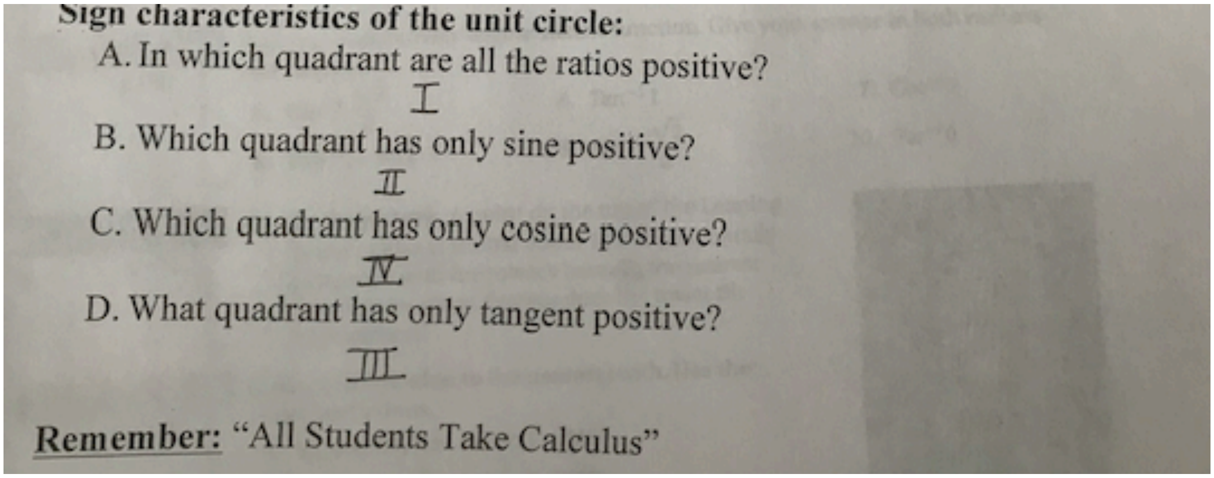
But any angle that is coterminal with either of these angles would also have that same sine value. Recall that to find coterminal angles in terms of radians we can add or subtract 2π. In this case, my answer indicates that I could add or subtract 2π an infinite amount of times and still find angles that have the same y-value. This is why we add the "where n is an integer" to the answer.

This answer indicates an infinite amount of solutions.

Example #2: Find all possible values of $\tan^{-1} 1$.

$\frac{\pi}{4} + 2\pi n$
$\frac{5\pi}{4} + 2\pi n$
where n is an integer.

In this case, we're looking for ratios of y/x that would be equal to 1. In order for that ratio to equal 1, the y and x values must be the same. This happens twice in the circle, $\frac{\pi}{4}$ *and* $\frac{5\pi}{4}$ and it also happens at their infinite coterminal angles.


Sign characteristics of the unit circle:

A. In which quadrant are all the ratios positive?

I

B. Which quadrant has only sine positive?

II

C. Which quadrant has only cosine positive?

IV

D. What quadrant has only tangent positive?

III

Remember: "All Students Take Calculus"

Knowing which quadrants have positive/negative trig values can make it faster to find your angle. When we say "All Students Take Calculus", we are creating an acronym to describe which ratios are positive in each quadrant.

A- All trig ratios are positive in quadrant I
S – sine is positive in quadrant II
T – tangent is positive in quadrant III
C – cosine is positive in quadrant IV

Example #3: **Solve for θ given that sin θ = 0.4, for $-90° \le \theta \le 90°$.**

** This question is Calculator Friendly since 0.4 is not a y-value on the unit circle. To compute inverse trig values on the Calculator, first make sure that you are in the correct MODE. In this case, we want to be in degree mode since the problem says **$-90° \le \theta \le 90°$.**

Since $\sin\theta = 0.4$

$\theta = \sin^{-1}(0.4)$

On the Calculator push 2ND SINE (.4) to find the inverse value.

$\theta = 23.578^{o}$

Example #4: **Solve for θ given that tan θ = -2, for $-90° < \theta < 90°$.**

**This question is also Calculator Friendly since -2 is not a ratio of y/x for any angle on the Unit Circle.

Since $\tan\theta = -2$

$\theta = \tan^{-1}(-2)$

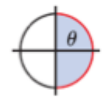
This time, we pish 2ND TAN (-2).

$\theta = -63.435^{o}$

Because more than one value of θ produces the same output value for a given trigonometric function, it is necessary to restrict the domain of each trigonometric function in order to define the inverse trigonometric functions.

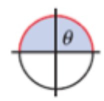
- Trigonometric functions with restricted domains are indicated with a capital letter.
- The domains of the Sine, Cosine, and Tangent functions are restricted as follows:

$\text{Sin}\theta = \sin\theta$ for $\{\theta | -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\}$



θ is restricted to Quadrants I and IV.

$\text{Cos}\theta = \cos\theta$ for $\{\theta | 0 \le \theta \le \pi\}$



θ is restricted to Quadrants I and II.

$\text{Tan}\theta = \tan\theta$ for $\{\theta | -\frac{\pi}{2} < \theta < \frac{\pi}{2}\}$

θ is restricted to Quadrants I and IV.

Example #5: NON-Calculator **Evaluate each inverse trigonometric function. Give your answer in radians.**

$\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{5\pi}{6}$

We look in Quad II since this cosine value is negative and Cos can only be in Quad I or II. The only angle in Quad II that has this x-value is $\frac{5\pi}{6}$

$\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$= \frac{-\pi}{4}$

We look in Quad IV since this sine value is negative and Sin can only be in Quad I or IV. The only angle in Quad IV that has this x-value is $\frac{7\pi}{4}$, but $\frac{7\pi}{4}$ is not between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$. So we use coterminal angles and subtract 2π to obtain $\frac{-\pi}{4}$

$Tan^{-1}(-\sqrt{3})$

$= \frac{-\pi}{3}$

We look in Quad IV since this tangent value is negative and Tan can only be in Quad I or IV. We need to look for an angle where the ratio of y/x is equal to $-\sqrt{3}$. This would mean that the y-value needs to involve $\frac{\sqrt{3}}{2}$ and the x-value needs to involve $\frac{1}{2}$. The only angle in Quad IV that has this x-value is $\frac{5\pi}{3}$, but $\frac{5\pi}{3}$ is not between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$. So we use coterminal angles and subtract 2π to obtain $\frac{-\pi}{3}$

$\text{Sin}^{-1}\frac{3}{2}$ NOT POSSIBLE, since the highest y-value in the unit circle is 1.

Write about it:

Explain the difference between $\tan^{-1}a$ and $\text{Tan}^{-1}a$.

$\tan^{-1}a$ has an infinite number of solutions because we must also consider coterminal angles, while $\text{Tan}^{-1}a$ has exactly one solution that is either in quadrant I or IV, but also between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$