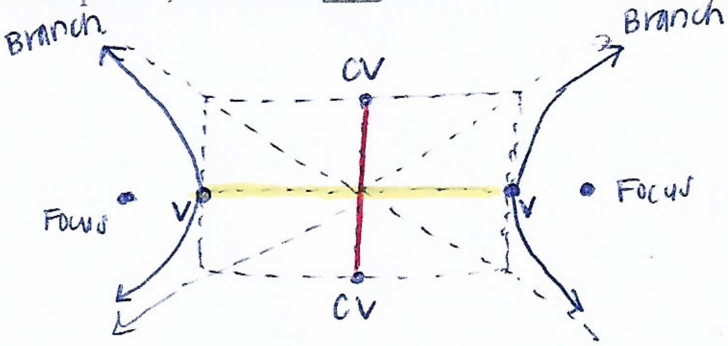
Section 10.4

Hyperbolas

A **hyperbola** is a set of points in a plane such that the difference of the distances from points on the hyperbola to fixed points, called the **foci**, is constant.



★ Not called major/minor since there is NO Restriction on which is larger.

- A hyperbola contains two symmetrical parts called **branches**.
- A hyperbola also has two axes of symmetry.
  - The **transverse axis** contains the vertices and, if it were extended, the foci of the hyperbola.
  - The **conjugate axis** separates the two branches of the hyperbola.
    - The transverse axis is NOT ALWAYS longer than the conjugate axis.
- The **vertices of a hyperbola** are the endpoints of the transverse axis.
- The **co-vertices of a hyperbola** are the endpoints of the conjugate axis.

Assume $c = \sqrt{a^2 + b^2}$ → New foci formula!

** a does not have to be larger than b

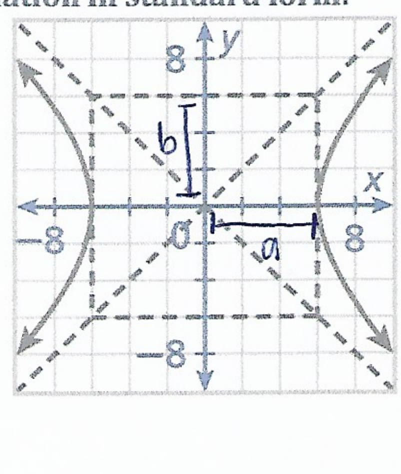
★ For hyperbolas, we identify the "a" as being the number under the first term. "b" is under the term getting subtracted.

Standard Form for the Equation of a Hyperbola Center at (0, 0)

TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Co-vertices	$(0, b), (0, -b)$	$(b, 0), (-b, 0)$
Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

Example #2: **Write the equation in standard form.**

★ horizontal since branches open horizontally.



$a = 6 \quad b = 6$

$$\boxed{\frac{x^2}{36} - \frac{y^2}{36} = 1}$$

Example #3: ★Horizontal since vertex changed in x

Write the equation in standard form of the hyperbola with center at the origin, vertex (4, 0), and focus (10, 0). $a = 4$ $c = 10$

$c = \sqrt{a^2 + b^2}$

$10 = \sqrt{16 + b^2}$

$100 = 16 + b^2$

$b^2 = 84$

$$\boxed{\frac{x^2}{16} - \frac{y^2}{84} = 1}$$

You try: **Vertex (0, 9), co-vertex (7, 0), Center at origin.**

$$\frac{y^2}{81} - \frac{x^2}{49} = 1$$

Example #4: **Find the vertices, co-vertices, foci, and asymptotes of the hyperbola, and then graph.**

$$\frac{x^2}{16} - \frac{y^2}{36} = 1$$

$a = 4$ $b = 6$

vertices: $(4, 0), (-4, 0)$

co-vertices: $(0, 6), (0, -6)$

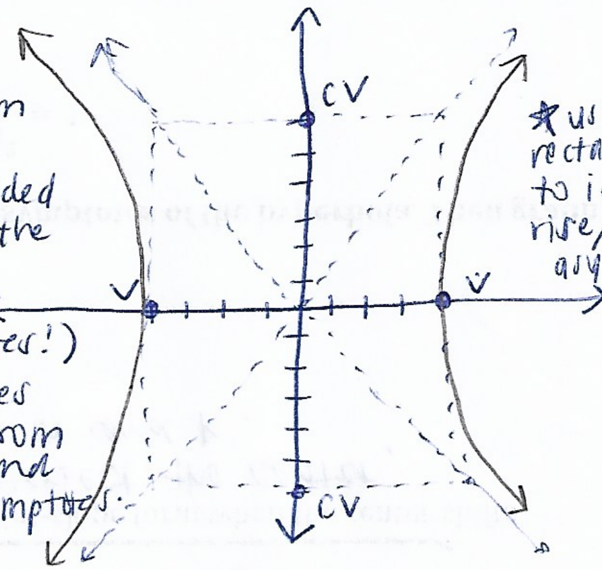
$c = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$

foci: $(2\sqrt{13}, 0), (-2\sqrt{13}, 0)$

★can be easier to find the asymptotes after graphing.

Asymptotes: $y = \pm \frac{3}{2}x$

① Plot vertices + co-vertices
② Connect to form a rectangle.
③ Draw the extended diagonals of the rectangle (These are the asymptotes!)
④ Draw branches extending from vertices and hug the asymptotes.



★use the rectangle to identify rise/run of asymptotes.

Standard form of a hyperbola:

$$\boxed{\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \text{ or } \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1}$$

center is (h, k) and a is first!

Example #5: **Identify the center, vertices, co-vertices the length of transverse axis, and the length of conjugate axis.**

$$\frac{(x-3)^2}{9} - \frac{(y+5)^2}{49} = 1$$

$a = 3$ $b = 7$

center: $(3, -5)$

vertices: $(0, -5), (6, -5)$

Covertices: $(3, 2), (3, -12)$

Length of transverse: $2a$

$2(3) = \boxed{6}$

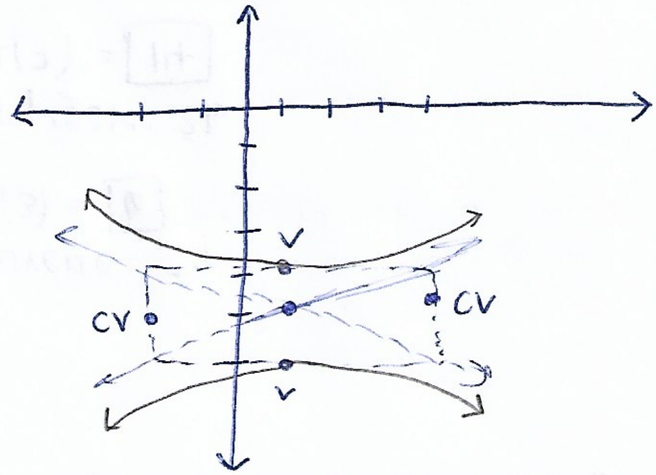
Length of conjugate: $2b$

$2(7) = \boxed{14}$

Example #6: **Find the center, vertices, co-vertices, foci, and asymptotes of the hyperbola. Then graph.**

$$\frac{(y+5)^2}{1} - \frac{(x-1)^2}{9} = 1$$

$a = 1$ $\quad b = 3$



Center: $(1, -5)$

vertices: $(1, -6), (1, -4)$

Covertices: $(-2, -5), (4, -5)$

$c = \sqrt{1+9} = \sqrt{10}$

foci: $(1, -5+\sqrt{10}), (1, -5-\sqrt{10})$

Asymptotes:

$$y + 5 = \pm \frac{1}{3}(x-1)$$

	Horizontal	Vertical
Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$

- Notice how the asymptote equations look like point-slope form when the center shifts

* The asymptotes should both intersect the center, which is why the formulas use h and k.

You try: **Find the center, vertices, co-vertices, foci, and asymptotes of the hyperbola. Then graph.**

* center is always (h, k) even when the y is first!

$$\frac{(y-2)^2}{16} - \frac{(x+2)^2}{25} = 1$$

center: $(-2, 2)$