## Hyperbolas

A hyperbola is a set of points in a plane such that the difference of the distances from points on the hyperbola to fixed points, called the foci, is constant.
$>$ A hyperbola contains two symmetrical parts called branches.
$>$ A hyperbola also has two axes of symmetry.

- The transverse axis contains the vertices and, if it were extended, the foci of the hyperbola.
- The conjugate axis separates the two branches of the hyperbola.
- The transverse axis is NOT ALWAYS longer than the conjugate axis.
$>$ The vertices of a hyperbola are the endpoints of the transverse axis.
$>$ The co-vertices of a hyperbola are the endpoints of the conjugate axis.

$$
\text { Assume } c=\sqrt{a^{2}+b^{2}}
$$

** a does not have to be larger than b
Standard Form for the Equation of a Hyperbola Center at (0, 0)

| TRANSVERSE AXIS | HORIZONTAL | VERTICAL |
| :--- | :---: | :---: |
| Equation | $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ | $\frac{y^{2}}{a^{2}}-\frac{x^{2}}{b^{2}}=1$ |
| Vertices | $(a, 0),(-a, 0)$ | $(0, a),(0,-a)$ |
| Foci | $(c, 0),(-c, 0)$ | $(0, c),(0,-c)$ |
| Co-vertices | $(0, b),(0,-b)$ | $(b, 0),(-b, 0)$ |
| Asymptotes | $y= \pm \frac{b}{a} x$ | $y= \pm \frac{a}{b} x$ |

Example \#2: Write the equation in standard form.


Example \#3:
Write the equation in standard form of the hyperbola with center at the origin, vertex $(4,0)$, and focus $(10,0)$.

You try: Vertex $(0,9)$, co-vertex $(7,0)$, Center at origin.

Example \#4: Find the vertices, co-vertices, foci, and asymptotes of the hyperbola, and then graph.

$$
\frac{x^{2}}{16}-\frac{y^{2}}{36}=1
$$

Standard form of a hyperbola:

$$
\frac{(x-h)^{2}}{a^{2}}-\frac{(y-k)^{2}}{b^{2}}=1 \text { or } \frac{(y-k)^{2}}{a^{2}}-\frac{(x-h)^{2}}{b^{2}}=1
$$

Example \#5: Identify the center, vertices, co-vertices the length of transverse axis, and the length of conjugate axis.

$$
\frac{(x-3)^{2}}{9}-\frac{(y+5)^{2}}{49}=1
$$

Example \#6: Find the center, vertices, co-vertices, foci, and asymptotes of the hyperbola. Then graph.

$$
\frac{(y+5)^{2}}{1}-\frac{(x-1)^{2}}{9}=1
$$

|  | Horizontal | $\underline{\text { Vertical }}$ |
| :--- | :--- | :--- |
| Asymptotes | $y-k= \pm \frac{b}{a}(x-h)$ | $y-k= \pm \frac{a}{b}(x-h)$ |

- Notice how the asymptote equations look like point-slope form when the center shifts

You try: Find the center, vertices, co-vertices, foci, and asymptotes of the hyperbola. Then graph.

$$
\frac{(y-2)^{2}}{16}-\frac{(x+2)^{2}}{25}=1
$$

