Practice for AP Test

1. $\int_1^2 (4x^3 - 6x)\, dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

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$\underline{\underline{C}}$ $\quad x^4 - 3x^2 \Big]_1^2$

$(16 - 12) - (1 - 3)$

$4 \quad + 2 \quad = 6$

---

2. If $f(x) = x\sqrt{2x - 3}$, then $f'(x) =$

(A) $\dfrac{3x - 3}{\sqrt{2x - 3}}$

(B) $\dfrac{x}{\sqrt{2x - 3}}$

(C) $\dfrac{1}{\sqrt{2x - 3}}$

(D) $\dfrac{-x + 3}{\sqrt{2x - 3}}$

(E) $\dfrac{5x - 6}{2\sqrt{2x - 3}}$

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$\underline{\underline{A}}$ $\quad f(x) = x(2x-3)^{1/2}$

$f'(x) = x \cdot \frac{1}{2}(2x-3)^{-1/2} \cdot 2 + (2x-3)^{1/2} \cdot 1$

$f'(x) = (2x-3)^{-1/2}(x + 2x - 3)$

$f'(x) = \dfrac{3x-3}{\sqrt{2x-3}}$

3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x) + 5)\,dx =$

(A) $a + 2b + 5$ (B) $5b - 5a$ (C) $7b - 4a$ (D) $7b - 5a$ (E) $7b - 6a$

---

$\underline{\underline{C}}$

$\int_a^b f(x)\,dx = a + 2b$

$F(x)\Big]_a^b = a + 2b$

$F(b) - F(a) = a + 2b$

$\int_a^b f(x) + 5 \; dx$

$= F(x) + 5x \Big]_a^b$

$= (F(b) + 5b) - (F(a) + 5a)$

$= \underbrace{F(b) - F(a)}_{} + 5b - 5a$

$= a + 2b + 5b - 5a$

$= 7b - 4a$

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4. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) $-1$ (D $-3$ E) $-5$

---

$\underline{\underline{D}}$

$f(x) = -x^3 + x + x^{-1}$

$f'(x) = -3x^2 + 1 - 1x^{-2}$

$= -3x^2 + 1 - \frac{1}{x^2}$

$f'(-1) = -3 + 1 - 1$

$f'(-1) = -3$

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

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$\underline{\underline{E}}$

$y' = 12x^3 - 48x^2 + 48x$

$y'' = 36x^2 - 96x + 48$

$36x^2 - 96x + 48 = 0$

$12(3x^2 - 8x + 4) = 0$

$12(3x-2)(x-2) = 0$

$3x - 2 = 0 \qquad x - 2 = 0$

$x = \frac{2}{3} \qquad x = 2$

$+ \qquad - \qquad +$

$\frac{2}{3} \qquad 2$

$x = 0 \qquad x = 1 \qquad x = 3$

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6. $\frac{1}{2} \int e^{\frac{t}{2}}\, dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

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$\underline{\underline{C}}$

$\frac{1}{2} \int e^{\frac{t}{2}}\, dt \qquad u = \frac{1}{2}t$

$\frac{1}{2} \int e^{\frac{1}{2}t}\, dt \qquad du = \frac{1}{2}\, dt$

$\frac{2}{1} \cdot \frac{1}{2} \int e^{\frac{1}{2}t} \frac{1}{2}\, dt$

$\int e^{u}\, du = e^{u} = e^{\frac{1}{2}t}$

7. $\dfrac{d}{dx}\cos^2(x^3) =$

(A) $6x^2\sin(x^3)\cos(x^3)$

(B) $6x^2\cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

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$\underline{\underline{D}}$ $\dfrac{d}{dx}\left[\cos(x^3)\right]^2$

$= 2\left[\cos(x^3)\right]^1\left(-\sin(x^3)\right)\cdot 3x^2$

$= -6x^2\cos(x^3)\sin(x^3)$

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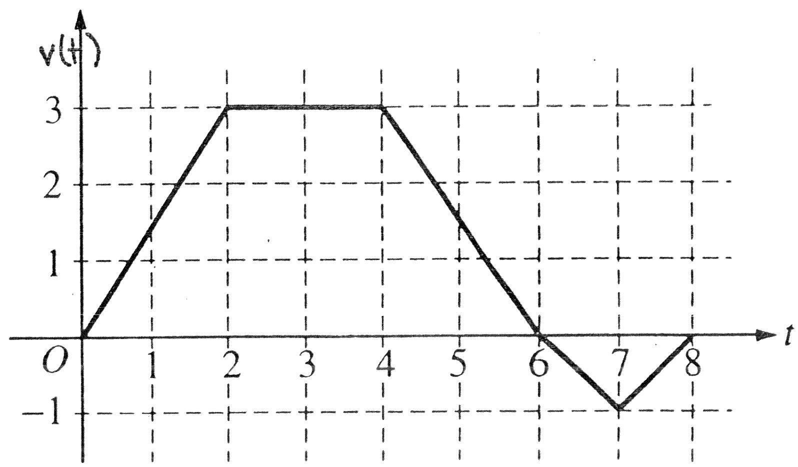
8. At what value of $t$ does the bug change direction?




A bug begins to crawl up a vertical wire at time $t = 0$. The velocity $v$ of the bug at time $t$, $0 \le t \le 8$, is given by the function whose graph is shown above.

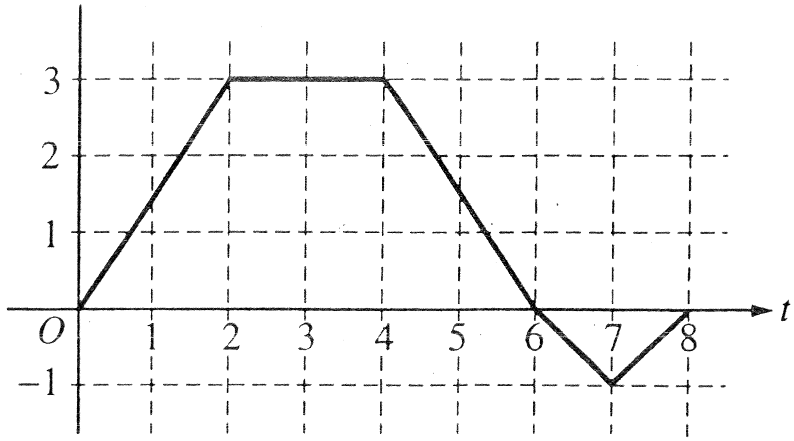
(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

---

$\underline{\underline{C}}$

At 1 min velocity $1\frac{1}{2}$
2 min " 3
3 min " 3
4 min " 3

At 5 min velocity $1\frac{1}{2}$
6 min 0 stop--d
7 min −1 Going [illegible]
8 min 0




9. What is the total distance the bug traveled from $t = 0$ to $t = 8$ ?

(A) 14 (B) 13 (C) 11

(D) 8 (E) 6

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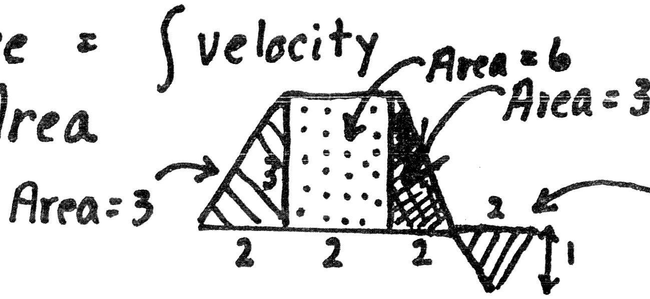
$\underline{\underline{B}}$ Distance = $\int$ velocity

Find Area




$\underbrace{3+6+3}_{\text{up}}+\underbrace{1}_{\text{down}}=13$

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10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

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slope of tangent = $m = y' = -\sin(2x)\cdot 2$

$\underline{\underline{E}}$

$m = y' = -2\sin(2x)$

$m = y' = -2\sin(2\cdot \frac{\pi}{4})$

$m = y' = -2\sin(\frac{\pi}{2})$

$m = y' = -2\cdot 1$

$m = -2$

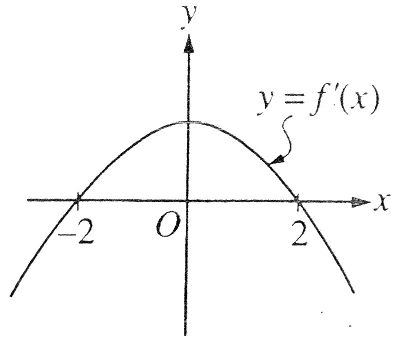
$y = \cos(2x)$

$y = \cos(2\cdot \frac{\pi}{4})$

$y = \cos\frac{\pi}{2}$

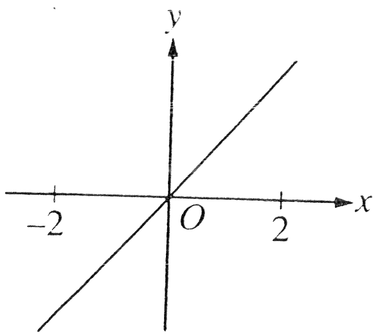
$y = 0$ $x = \frac{\pi}{4}$

$y - 0 = -2(x - \frac{\pi}{4})$

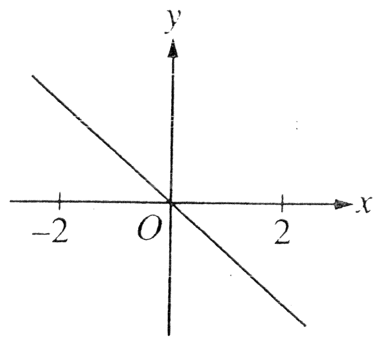



11. The graph of the derivative of $f$ is shown in the figure above. Which of the following could be the graph of $f$?

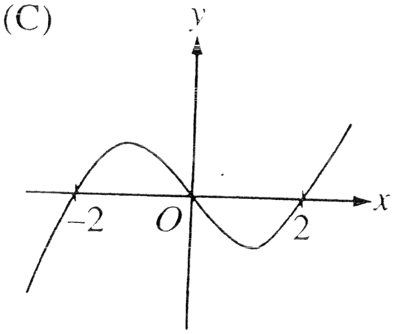
(A)



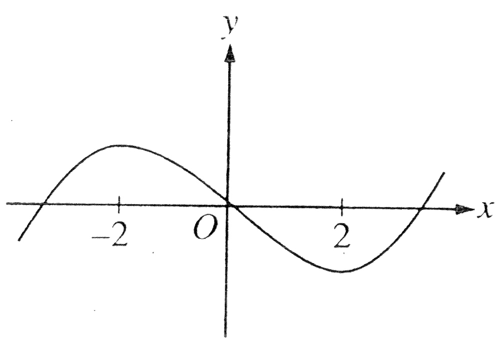

(B)



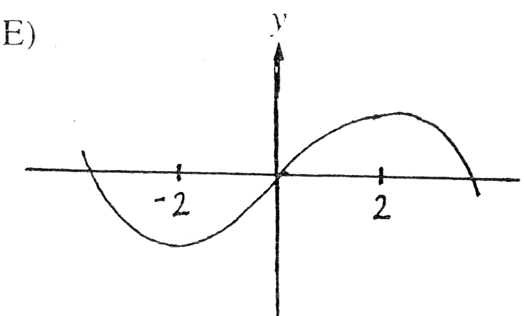

(C)




(D)




(E)

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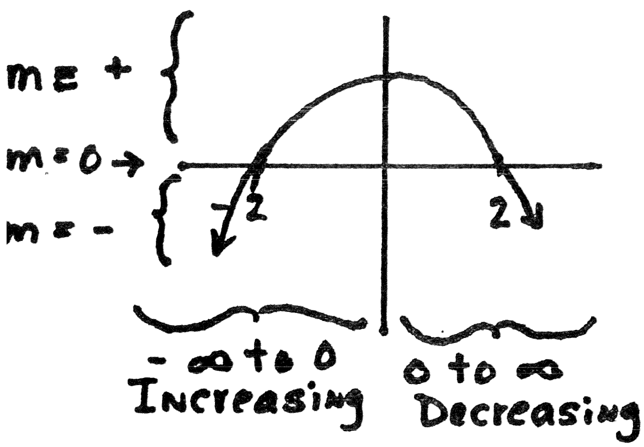
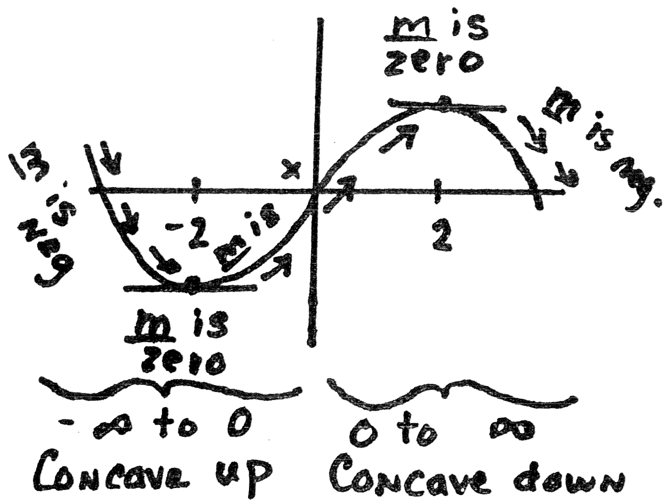
$\underline{\underline{E}}$

$f'(x)$ = slope of tangent lines




$f(x)$



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14. Let $f$ be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of $f$ at $x = 3$ is used to find an approximation to a zero of $f$, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

---

C $f'(3) = 5$ means slope at $x = 3$ is 5

"tangent line" at pt $(3,2)$ $m = 5$

$y - 2 = 5(x - 3)$

"to a zero of f" means x-intercept where $y = 0$

$$y - 2 = 5(x - 3)$$
$$0 - 2 = 5(x - 3)$$
$$-2 = 5x - 15$$
$$13 = 5x$$
$$2.6 = x$$

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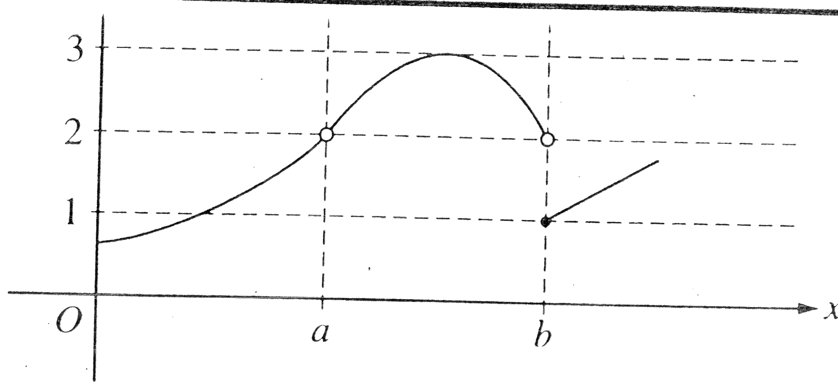


15. The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

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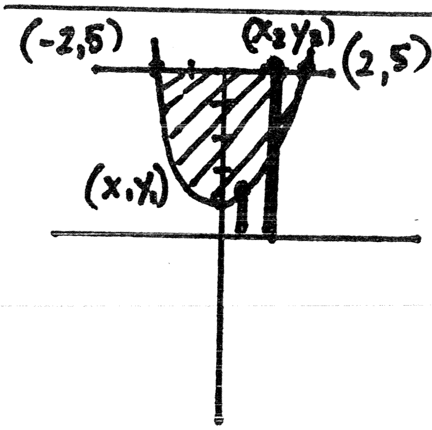
B Remember $\lim_{x \to a}$ means $\lim_{x \to a^+} = \lim_{x \to a^-}$

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8

D




$$\text{Area} = 2\int_0^2 y_2 - y_1\,dx$$
$$= 2\int_0^2 5-(x^2+1)\,dx$$
$$= 2\int_0^2 4-x^2\,dx$$
$$= 2\cdot 4x - \tfrac{1}{3}x^3\Big]_0^2$$
$$= 2\left(8-\tfrac{8}{3}\right)$$
$$= 2\left(\tfrac{16}{3}\right)$$
$$= \frac{32}{3}$$

17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point (4, 3) ?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

A

$$x^2 + y^2 = 25$$
$$2x + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

2Nd Derivative = ?
(Quotient Rule)

$$\frac{d^2y}{dx^2} = \frac{y(-1)-(-x)\frac{dy}{dx}}{y^2}$$
$$= \frac{3(-1)-(-4)(-\tfrac{4}{3})}{9}$$
$$= \frac{-3-\tfrac{16}{3}}{9}$$
$$= -\frac{25}{3}\cdot\frac{1}{9}$$
$$= -\frac{25}{27}$$

18. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e - 1$

(D) $e$ (E) $e + 1$

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$\underline{\underline{C}}$

$u = \tan x$
$du = \sec^2 x$

$\int e^{\tan x} \cdot \underbrace{\sec^2 x\,dx}_{du}$
$\int e^{u}$

$= e^{u}$
$= e^{\tan x}\Big]_0^{\pi/4}$
$= e^{\tan \pi/4} - e^{\tan 0\pi}$
$= e^1 - e^0$
$= e^1 - 1$

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19. If $f(x) = \ln|x^2 - 1|$, then $f'(x) =$

(A) $\left|\dfrac{2x}{x^2 - 1}\right|$

(B) $\dfrac{2x}{|x^2 - 1|}$

(C) $\dfrac{2|x|}{x^2 - 1}$

(D) $\dfrac{2x}{x^2 - 1}$

(E) $\dfrac{1}{x^2 - 1}$

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$\underline{\underline{D}}$

$f(x) = \ln|x^2 - 1|$
$f'(x) = \dfrac{1}{x^2 - 1} \cdot 2x$
$= \dfrac{2x}{x^2 - 1}$

20. The average value of $\cos x$ on the interval $[-3, 5]$ is

(A) $\dfrac{\sin 5 - \sin 3}{8}$

(B) $\dfrac{\sin 5 - \sin 3}{2}$

(C) $\dfrac{\sin 3 - \sin 5}{2}$

(D) $\dfrac{\sin 3 + \sin 5}{2}$

(E) $\dfrac{\sin 3 + \sin 5}{8}$

---

$\underline{\underline{E}}$

Area Under Curve = length · width

$$\int_{-3}^{5} \cos x \, dx = (5 - {}^{-}3) \cdot f(z)$$

$$\sin x \Big]_{-3}^{5} = 8 \cdot f(z)$$

$$\sin 5 - \sin(-3) = 8 \cdot f(z)$$

$\boxed{\text{Recall : } \sin(-x) = -\sin x}$

$$\frac{\sin 5 + \sin 3}{8} = f(z)$$

---

21. $\displaystyle \lim_{x \to 1} \frac{x}{\ln x}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $e$ (E) nonexistent

---

$\underline{\underline{E}}$ $\displaystyle \lim_{x \to 1} \frac{x}{\ln x} = \frac{1}{0}$

Nonexistent

22. What are all values of $x$ for which the function $f$ defined by

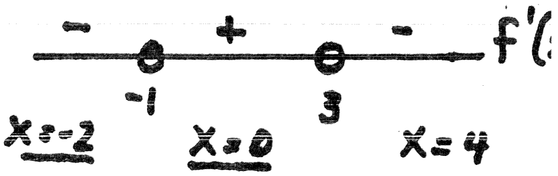
$f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of $x$.
(B) $x < -1$ and $x > 3$
(C) $-3 < x < 1$
(D) $-1 < x < 3$
(E) All values of $x$

---

$\underline{\underline{D}}$

$$f'(x) = (x^2-3)(-e^{-x}) + e^{-x}(2x)$$
$$= -e^{-x}[x^2-3-2x]$$
$$= -e^{-x}[x^2-2x-3]$$
$$-\frac{1}{e^x} \neq 0 \qquad (x-3)(x+1)=0$$
$$x=3 \quad x=-1$$




increasing $(-1,3)$

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23. If the region enclosed by the $y$-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the $y$-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) $\pi$

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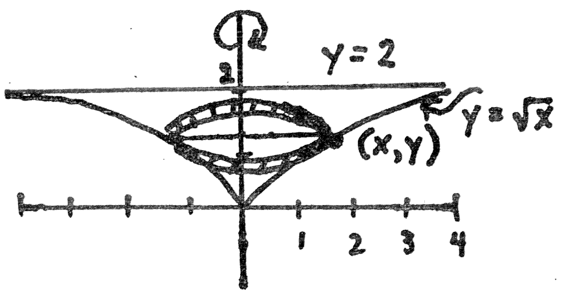
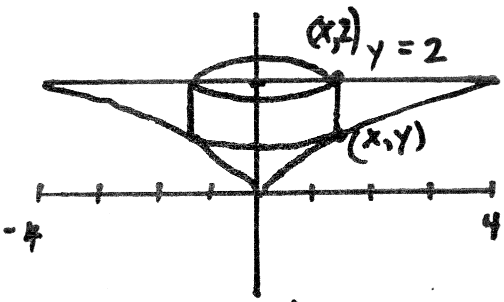
$\underline{\underline{A}}$




Disk Method

$$\int_0^2 \pi r^2\,dy$$
$$\pi\int x^2\,dy$$
$$\pi\int_0^2 y^4\,dy = \pi\cdot\frac{1}{5}y^5\Big]_0^2$$
$$= \frac{32\pi}{5}$$

Shell Method

$$\int_0^4 2\pi r\,l\,w$$
$$\int_0^4 2\pi x\,(2-y)\,dx$$
$$2\pi\int x(2-\sqrt{x})\,dx$$
$$2\pi\int 2x - x^{3/2}\,dx$$
$$2\pi\cdot x^2 - \tfrac{2}{5}x^{5/2}\Big]_0^4 = \frac{32\pi}{5}$$

24. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}} + \sqrt{\frac{2}{50}} + \sqrt{\frac{3}{50}} + \ldots + \sqrt{\frac{50}{50}}\right)$

is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

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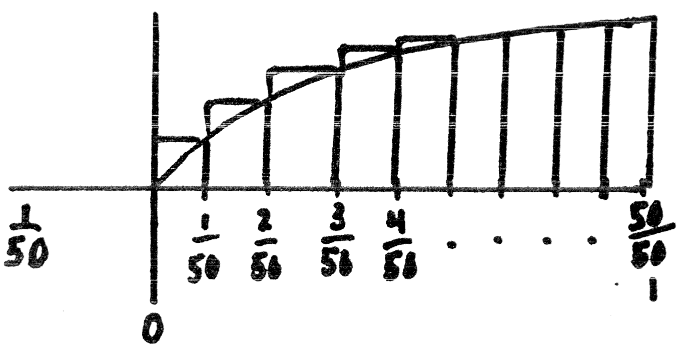
$\underline{\underline{B}}$

width $= \frac{1}{50}$




Area 1st rect $= w \cdot l$

$= \frac{1}{50} \cdot \sqrt{\frac{1}{50}}$

Area 2nd rect $= \frac{1}{50} \cdot \sqrt{\frac{2}{50}}$

$\int_0^1 \sqrt{x}\,dx$

$l \cdot$ width

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25. $\int x \sin(2x)\,dx =$

(A) $-\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + C$

(B) $-\frac{x}{2}\cos(2x) - \frac{1}{4}\sin(2x) + C$

(C) $\frac{x}{2}\cos(2x) - \frac{1}{4}\sin(2x) + C$

(D) $-2x\cos(2x) + \sin(2x) + C$

(E) $-2x\cos(2x) - 4\sin(2x) + C$

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$\underline{\underline{A}}$ $\int x \sin(2x)\,dx$

$u = x \qquad v = -\frac{1}{2}\cos 2x$

$du = dx \qquad dv = \sin 2x\,dx$

Integration By Parts

$\int u\,dv = uv - \int v\,du$

$= -\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x\,dx$

$= -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x\,dx \qquad \begin{cases} u = 2x \\ du = \frac{1}{2}\,2dx \end{cases}$

$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\,dx + C$

26. If $f(x) = \frac{e^{2x}}{2x}$, then $f'(x) =$

*

(A) 1

(B) $\frac{e^{2x}(1-2x)}{2x^2}$

(C) $e^{2x}$

(D) $\frac{e^{2x}(2x+1)}{x^2}$

(E) $\frac{e^{2x}(2x-1)}{2x^2}$

---

E

$$f'(x) = \frac{(2x)(2e^{2x}) - (e^{2x})(2)}{4x^2}$$

$$= \frac{2e^{2x}(2x-1)}{4x^2}$$

$$= \frac{e^{2x}(2x-1)}{2x^2}$$

---

27. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

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(A) $-1.58$ (B) $-1.63$ (C) $-1.67$ (D) $-1.89$ (E) $-2.33$

---

D

$y' = 3x^2 + 12x + 7 + 2\sin x$ Zeros are Max or Min Pts

$y'' = 6x + 12 + 2\cos x$ Zeros are inflection Pts. where concavity changes

Graph y" on calculator and look where it crosses x-axis

$-1.89$

28.
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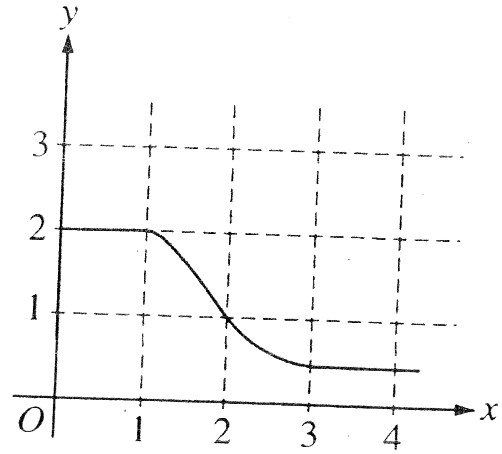



The graph of $f$ is shown in the figure above.

If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then

$F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

D

$\int_1^3 f(x) = 2.3$

Area under curve $\int_1^3 = 2.3$

$F(x)$ = Integral

$F'(x)$ = original function, $f(x)$

$\int f(x)\,dx = F(x)$

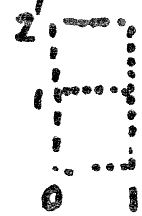
$\int_1^3 f(x)\,dx = F(3) - F(1) = 2.3$

$\int_0^1 f(x)\,dx = 2$ (from picture)

$\int_0^3 = \int_0^1 + \int_1^3$

$= 2 + 2.3$

$= 4.3$



29.

Let $f$ be a function such that $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true

I. $f$ is continuous at $x = 2$.
II. $f$ is differentiable at $x = 2$.
III. The derivative of $f$ is continuous at $x = 2$.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

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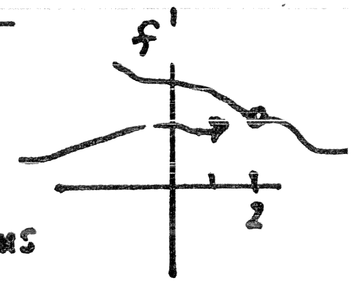
C Means slope of tangent line $f'(2) = 5$

$\lim_{x \to 2^+} = 5$ $\lim_{x \to 2^-} = 5$

$f'$ does not have to be continuous




On p 134 " For a function to be differentiable at a pt. The function must be continuous there.

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Let $f$ be the function given by $f(x) = 2e^{4x^2}$. For what value of $x$ is the slope of the line tangent to the graph of $f$ at $(x, f(x))$ equal to 3 ?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

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A

$f'(x) = 2e^{4x^2} \cdot 8x$

$f'(x) = 16xe^{4x^2}$

$m = 16xe^{4x^2}$

$3 = 16xe^{4x^2}$

$\frac{3}{16} = xe^{4x^2}$

$.1875 = xe^{4x^2}$

Use Calculator

$y = .1875$

$y = xe^{4x^2}$

Find Intersection

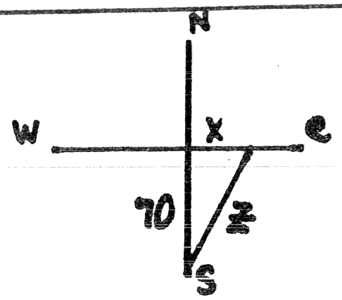
$(.168, .1875)$

31. A railroad track and a road cross at right angles. An observer stands on the road
*
70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

---

<u>A</u>




Given: $\frac{dx}{dt} = 60$ m/s

Find: $\frac{dz}{dt}$ when $x = 240$ m $(60\text{ m/sec} \times 4)$

EQ: $x^2 + 70^2 = z^2$

$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$

$2(240)(60) = 2(250)\frac{dz}{dt}$

$57.6 = \frac{dz}{dt}$

$70^2 + 240^2 = z^2$

$250 = z$

---

32. If $y = 2x - 8$, what is the minimum value of the product $xy$?
*

(A) −16 (B) −8 (C) −4 (D) 0 (E) 2

---

<u>B</u>

Minimize Product xy

$f(x) = x \cdot y$

$f(x) = x(2x-8)$

$f(x) = 2x^2 - 8x$

$f'(x) = 4x - 8$

$4x - 8 = 0$

$x = 2$

$-$ 2 $+$ $f'(x)$

$\underline{x=0}$ $\underline{x=3}$

Minimum Value occurs at $x = 2$

$y = 2x - 8$

$y = 2 \cdot 2 - 8$

$y = -4$

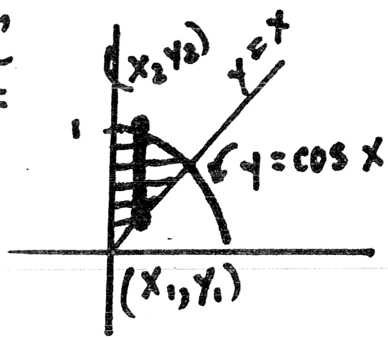
Minimum Value of $xy =$

$2 \cdot -4 = -$

33. What is the area of the region in the first quadrant enclosed by the graphs of
* $y = \cos x$, $y = x$, and the $y$-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

C =




Area = $\int L \cdot W$

$= \int (Y_2 - Y_1) \cdot dx$

$= \int_0^{.723} (\cos x - x)\, dx$

Find intersection using calculator

$(.723, .723)$

$Y_1 = \cos X$

$Y_2 = X$

2nd Calc #7 $\int_0^{.723} \cos x = .6619$

Use Arrows - go to $Y_2 =$)

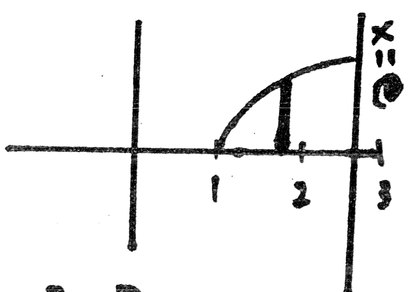
2nd Calc #7 $\int x = .26165$

$.66194 - .26165 = .4002$

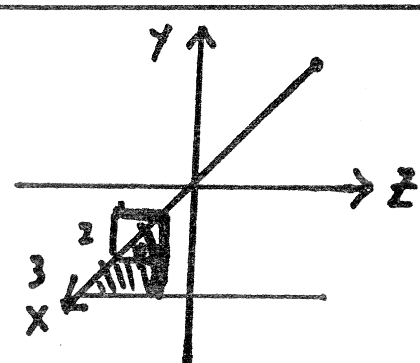
34. The base of a solid $S$ is the region enclosed by the graph of $y = \sqrt{\ln x}$,
* the line $x = e$, and the $x$-axis. If the cross sections of $S$ perpendicular to the $x$-axis are squares, then the volume of $S$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

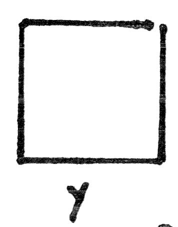
C =




2 - D




3 - D




$A = y^2$

$\int$ Area of Cross Section $\cdot\, dx$

$\int_1^e y^2\, dx$

$\int (\sqrt{\ln x})^2\, dx$

$\int_1^e \ln x\, dx$

$x \ln x - x \Big]_1^e$

$(e \ln e - e) - (1 \ln 1 - 1)$

$(e \cdot 1 - e) - (0 - 1)$

$0 + 1 = 1$

35. If the derivative of $f$ is given by $f'(x) = e^x - 3x^2$, at which of the following values of $x$ does $f$ have a relative maximum value?
*

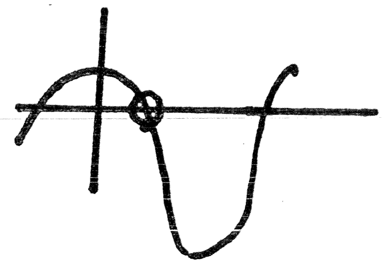
(A) $-0.46$ (B) $0.20$ (C) $0.91$ (D) $0.95$ (E) $3.73$

---

C $f'(x) = e^x - 3x^2$

Max Value occurs


From Calculator
$f'(x)$ looks like this



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36. Let $f(x) = \sqrt{x}$. If the rate of change of $f$ at $x = c$ is twice its rate of change at $x = 1$, then $c =$
*

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

---

A $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

$f'(c) = \frac{1}{2\sqrt{c}}$

$\frac{1}{2\sqrt{c}} = \underset{\text{twice}}{2} \cdot \frac{1}{2\sqrt{1}}$

$\frac{1}{2\sqrt{c}} = \frac{1}{1}$

$2\sqrt{c} = 1$

$\sqrt{c} = \frac{1}{2}$

$c = \frac{1}{4}$

---

37. At time $t \geq 0$, the acceleration of a particle moving on the $x$-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is $-2$. For what value of $t$ will the velocity of the particle be zero?
*

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

---

B $a(t) = t + \sin t$

$v(t) = \frac{1}{2}t^2 - \cos t + C$

$-2 = \frac{1}{2}(0)^2 - \cos(0) + C$

$-2 = 0 - 1 + C$

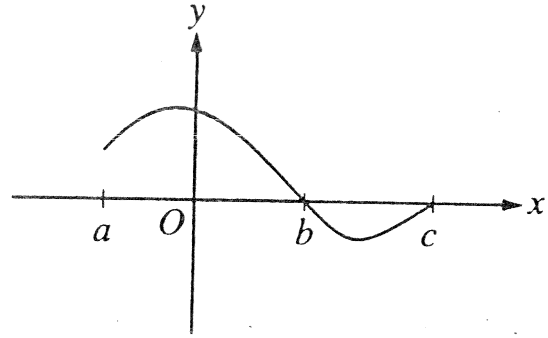
$-1 = C$

$v(t) = \frac{1}{2}t^2 - \cos t - 1$

Use calculator $t \geq 0$

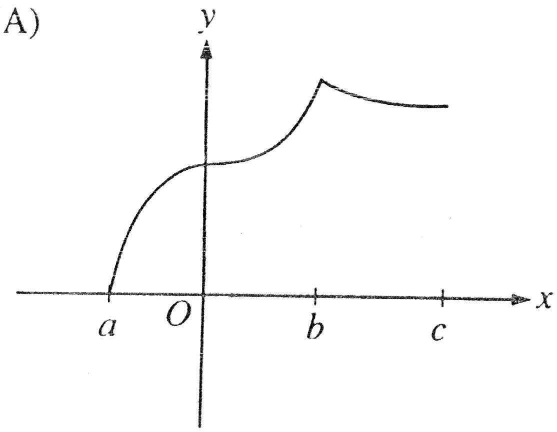
zero $= 1.478$

38.

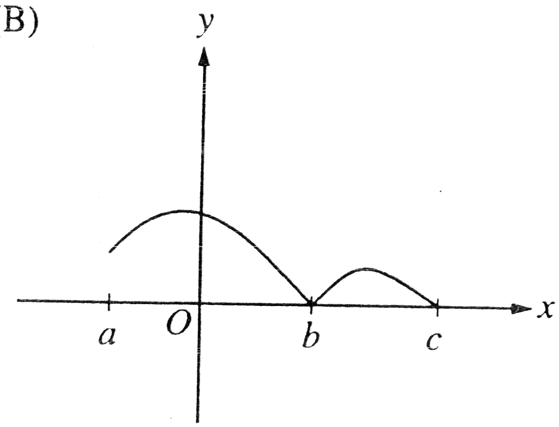



Let $f(x) = \int_a^x h(t)\, dt$, where $h$ has the graph shown above. Which of the following could graph of $f$ ?

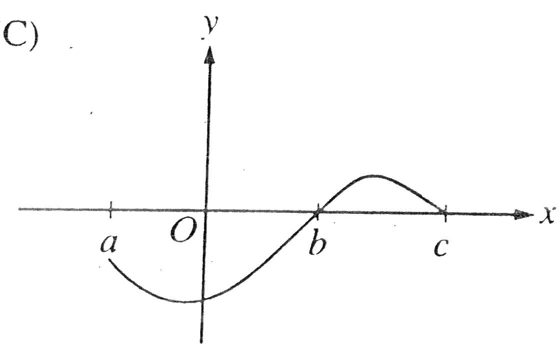
(A)



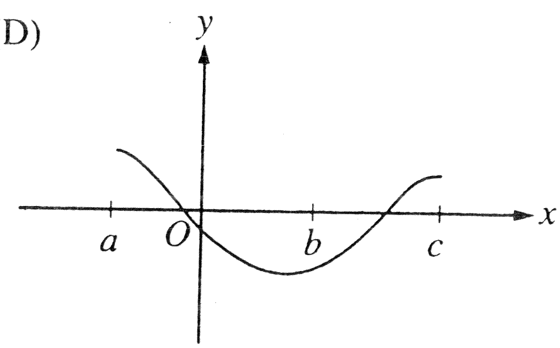

(B)



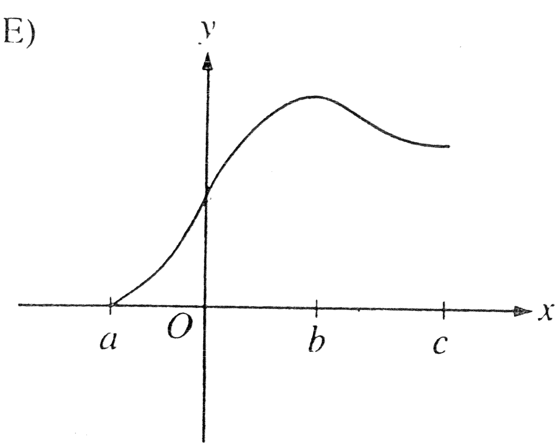

(C)




(D)




(E)



38. Answers

**E** $f(x) = \int_a^x h(t)\,dt$

h(x) is the graph of derivative

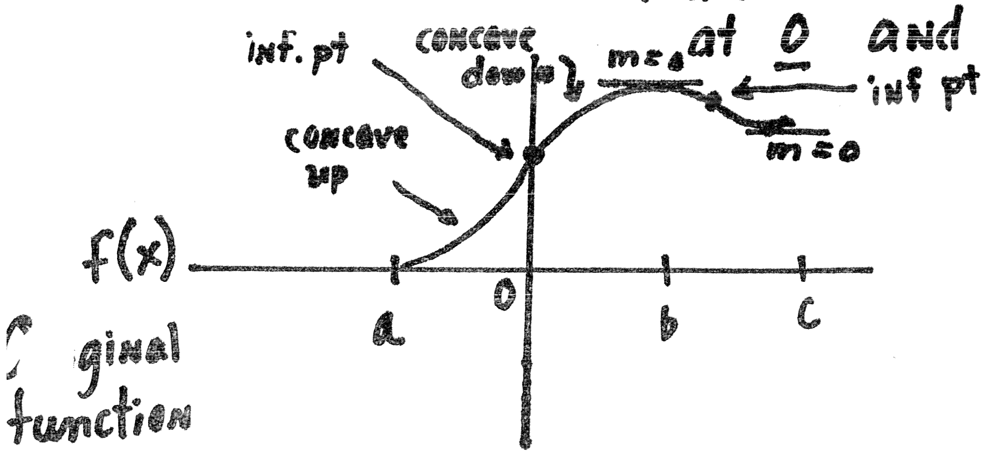
f(x) is original function

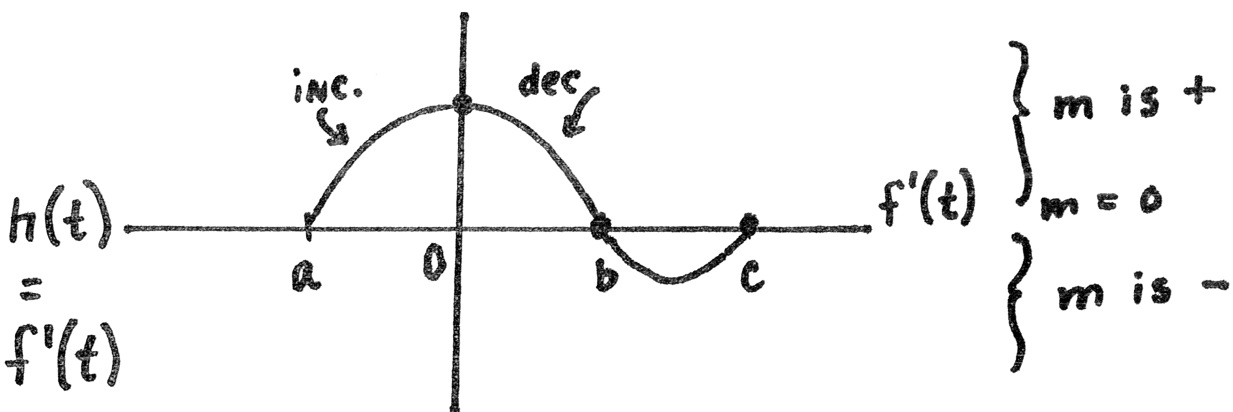
From graph: f(x) has a zero slope at $\underline{b}$ & $\underline{c}$

$\underline{b}$ is a max

$\underline{c}$ is a min

f(x) has an inflection pt at $\underline{0}$ and between $\underline{b}$ & $\underline{c}$




1st Derivative

39. *

| $x$ | 0 | 0.5 | 1.0 | 1.5 | 2.0 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 3 | 5 | 8 | 13 |

A table of values for a continuous function $f$ is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$ ?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

---

B $\int_0^2 f(x)dx = \frac{b-a}{2n}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$

$= \frac{2-0}{2\cdot 4}\left[3 + 2\cdot 3 + 2\cdot 5 + 2\cdot 8 + 13\right]$

$= \frac{1}{4}\left[3 + 6 + 10 + 16 + 13\right] =$

$= \frac{1}{4}\cdot 48 = 12$

---

40. Which of the following are antiderivatives of $f(x) = \sin x \cos x$? *

I. $F(x) = \frac{\sin^2 x}{2}$

II. $F(x) = \frac{\cos^2 x}{2}$

III. $F(x) = \frac{-\cos(2x)}{4}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

---

D F(x) is original function
F'(x) = f(x) is derivative

I. $F(x) = \frac{1}{2}(\sin x)^2$
$F'(x) = 2\cdot\frac{1}{2}\cdot\sin x\cos x$
$= 1\sin x\cos x$

~~II.~~ $F(x) = \frac{1}{2}(\cos x)^2$
$F'(x) = \frac{1}{2}\cdot 2\cdot\cos x\cdot(-\sin x)$
$= -\sin x\cos x$

III. $F(x) = -\frac{1}{4}\cos(2x)$
$= -\frac{1}{4}(-\sin(2x))(2)$
$= \frac{1}{2}\sin 2x = \frac{1}{2}(2\sin x\cos x)$
$= \sin x\cos x$

I + III only

41. $\displaystyle\int_0^1 \sqrt{x}(x+1)\,dx =$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

---

$\underline{\underline{C}}$ $\displaystyle\int_0^1 x^{3/2} + x^{1/2}\,dx$

$\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2}\Big]_0^1$

$\left(\frac{2}{5} + \frac{2}{3}\right) - (0+0)$

$\frac{6}{15} + \frac{10}{15} = \frac{16}{15}$

---

42. If $x = e^{2t}$ and $y = \sin(2t)$, then $\dfrac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\dfrac{e^{2t}}{\cos(2t)}$ (C) $\dfrac{\sin(2t)}{2e^{2t}}$ (D) $\dfrac{\cos(2t)}{2e^{2t}}$ (E) $\dfrac{\cos(2t)}{e^{2t}}$

---

$\underline{\underline{E}}$ $\dfrac{dx}{dt} = 2e^{2t}$ $\dfrac{dy}{dt} = 2\cos 2t$ $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\cancel{2}^{1}\cos 2t}{\cancel{2}_{1}e^{2t}}$

---

43. The function $f$ given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) $-1$ (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

---

$\underline{\underline{A}}$ $f'(x) = 15x^4 - 12x^2 - 3$

$= (3x^2-3)(5x^2+1)$

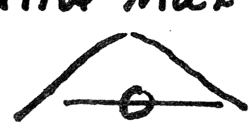
$x^2 = 1$ $x^2 = -\frac{1}{5}$

$x = \pm 1$ $x =$ imaginary #

$f'(x)$: $+$ | $-1$ | $-$ | $1$ | $+$

$x=-2$ $x=0$ $x=2$

Relative Max at $x=-1$



$-1$

44. $\frac{d}{dx}\left(x e^{\ln x^2}\right) =$

(A) $1 + 2x$ (B) $x + x^2$ (C) $3x^2$ (D) $x^3$ (E) $x^2 + x^3$

C

$e^{\ln_e x^2} = x^2$

$\frac{d}{dx}(x \cdot x^2) = \frac{d}{dx}(x^3) = 3x^2$

45. If $f(x) = (x - 1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

C

$f'(x) = \frac{3}{2}(x-1)^{1/2}(1) + \frac{1}{2}(e^{x-2})(1)$

$f'(2) = \frac{3}{2}(2-1)^{1/2}(1) + \frac{1}{2}(e^{2-2})(1)$

$= \frac{3}{2}(1)\cdot(1) + \frac{1}{2}\cdot e^0 \cdot 1$

$= 3/2 + 1/2$

$= 2$

46. The line normal to the curve $y = \sqrt{16 - x}$ at the point $(0, 4)$ has slope

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) $-8$

A

$y = (16-x)^{1/2}$

$y' =$ slope tangent $= \frac{1}{2}(16-x)^{-1/2}(-1)$

$y'(0) = \frac{1}{2}(16-0)^{-1/2}(-1)$

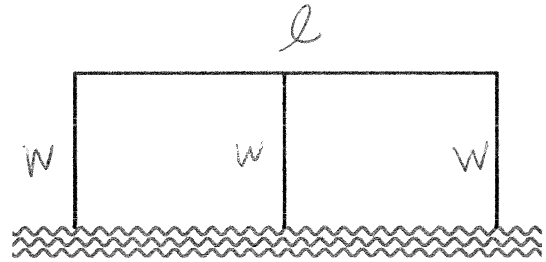
$y'(0) = \frac{-1}{2\sqrt{16}} = -\frac{1}{8}$

slope of tangent $= -\frac{1}{8}$

slope of normal $= +8$

**AP Calculus Review**

1.) A rancher wishes to build a rectangular pen adjacent to a river as shown in the diagram to the right. If the rancher has 600 meters of fencing to use, what is the maximum area that can be enclosed by the entire pen?



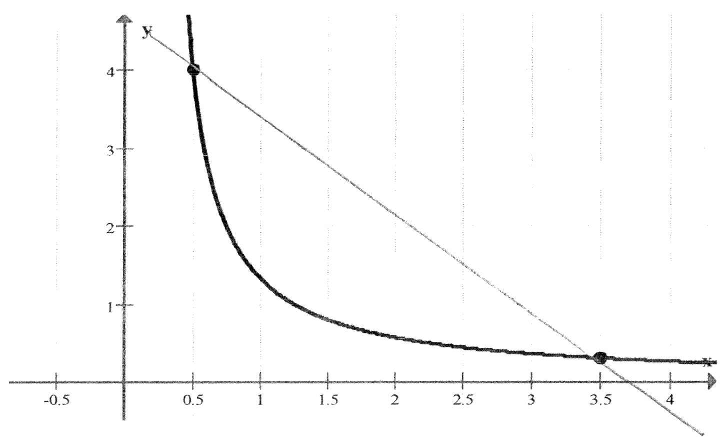

[a] $100m^2$ [b] $300m^2$ [c] $600m^2$ [d] $10000m^2$ [e] $30000m^2$

$P = l + 3w$ $A = lw$ $A = w(600 - 3w)$

$600 = l + 3w$ $A = 600w - 3w^2$

$l = 600 - 3w$ $A' = 600 - 6w$ $w = 100$

2.) Consider the function $y = f(x)$ shown in the graph to the right. Which of the following gives the best approximation for the value of $c$ guaranteed by the Mean Value Theorem on the interval $[0.5, 3.5]$?




[a] 0.5
[b] 0.75
[c] 1.25
[d] 2
[e] 3.5