2008 Calc AB Mult. Choice

1. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)} =$

$\lim_{x\to\infty} \frac{-2x^2}{x^2} =$

$\boxed{B}$ -2

2. $\int \frac{1}{x^2}\,dx =$

$\int x^{-2}\,dx =$

$-x^{-1}+C$

$\boxed{D}$

3. $f(x) = (x-1)(x^2+2)^3$

$f'(x) = (x-1)\cdot 3(x^2+2)^2\cdot 2x \; +$
$(x^2+2)^3\cdot 1$

$= (x^2+2)^2\left(6x(x-1) + (x^2+2)\right)$

$= (x^2+2)^2(6x^2-6x+x^2+2)$

$= (x^2+2)^2(7x^2-6x+2)$

$\boxed{D}$

4. $\int (\sin(2x) + \cos(2x))\,dx =$

$-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

$\boxed{B}$

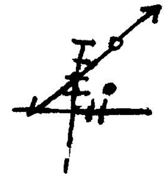
5. $\lim_{x\to 0} \frac{x^2(5x^2+8)}{x^2(3x^2-16)} =$

$\lim_{x\to 0} \frac{5x^2+8}{3x^2-16} =$

$-\frac{1}{2}$ $\boxed{A}$

6. $f(x) = \begin{cases} \frac{(x+2)(x-2)}{x-2}, & x\neq 2 \\ 1, & x=2 \end{cases}$

$= \begin{cases} x+2, & x\neq 2 \\ 1, & x=2 \end{cases}$



only I is true

$\boxed{A}$

7. Method 1

$x(t) = t^3+3t+C$

$2 = 0+0+C$

$x(t) = t^3+3t+2$

$x(1) = 1+3+2$

$= 6$

$\boxed{B}$

Method 2

$x(1) = 2 + \int_0^1 (3t^2+6t)\,dt$

$= 2 + (t^3+3t)\Big|_0^1$

$= 2 + (1+3) - 0$

$= 6$

8. $f(x) = \cos(3x)$

$f'(x) = -\sin(3x)\cdot 3$

$f'\left(\frac{\pi}{9}\right) = -3\sin\frac{\pi}{3}$

$= -3\cdot\frac{\sqrt{3}}{2}$

$\boxed{E}$

9. Method 1

$g'(x) = f(x)$

	-3		1		2
g'		$+$		$-$	

g has an absolute max. at $x = 1$

$\boxed{D}$

Method 2

Evaluate Geometrical Areas

$g(-3) = \int_{-2}^{-3} f(t)\,dt = -\frac{3}{2}$

$g(-2) = \int_{-2}^{-2} f(t)\,dt = 0$

$g(0) = \int_{-2}^{0} f(t)\,dt = 3$

$g(1) = \int_{-2}^{1} f(t)\,dt = 4$ (max.)

$g(2) = \int_{-2}^{2} f(t)\,dt = 3$

10. Since f is decreasing the Right Riemann sum is less than all the other choices.

$\boxed{C}$

11. The rel. max. on f is a zero on f' (where f' switches from pos. to neg.). The rel. min. on f is a zero on f' (where f' switches from neg. to pos.)

$\boxed{B}$

12. $f(x) = e^{2x^{-1}}$

$f'(x) = e^{2x^{-1}} \cdot -2x^{-2}$

$= \dfrac{-e^{2/x}}{x^2}$

$\boxed{D}$

13. $\frac{d}{dx}\left(f(\ln x)\right) =$

$\frac{d}{dx}\left((\ln x)^2 + 2\ln x\right) =$

$2\ln x \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} =$

$\dfrac{2\ln x + 2}{x}$ $\boxed{A}$

14. A, B, C may or may not be true

D may not be true because f'' may not change sign at $x = 1$

E must be true because f'' changes sign at least once in $(0, 2)$

$\boxed{E}$

15. $\int \frac{x}{x^2-4}\,dx$

$\frac{1}{2}\int \frac{2x}{x^2-4}\,dx$

$\frac{1}{2}\ln|x^2-4| + C$

$\boxed{C}$

16. $\sin(xy) = x$

$\cos(xy)(xy' + y) = 1$

$xy' + y = \dfrac{1}{\cos(xy)}$

$xy' = \dfrac{1}{\cos(xy)} - y$

$y' = \dfrac{1}{x\cos(xy)} - \dfrac{y}{x}$

$= \dfrac{1 - y\cos(xy)}{x\cos(xy)}$

$\boxed{D}$

17. $g'(x) = f(x)$

$g''(x) = f'(x)$

$f'(x) = 0$ at $x = 2, 5$

and g'' changes sign

$\boxed{C}$

18. $y = k - x$ has slope -1

$y = x^2 + 3x + 1 \rightarrow y' = 2x + 3$

$2x + 3 = -1$

$x = -2 \rightarrow y = 4 - 6 + 1$
$= -1$

$x + y = k$

$-2 - 1 = k$

$-3 = k$

$\boxed{A}$

19. $\displaystyle\lim_{x\to\infty} \frac{5 + 2^x}{1 - 2^x} =$

$\displaystyle\lim_{x\to\infty} \frac{2^x}{-2^x} = -1$

$\displaystyle\lim_{x\to-\infty} \frac{5 + 2^x}{1 - 2^x} = 5$

$\boxed{E}$

20. $f''(x) = x^2(x-3)(x-6)$

f'': $+$ | $+$ | $-$ | $+$ at 0, 3, 6

$\boxed{D}$

21. $v(t) = x'(t)$

$v'(t) = x''(t)$

v is increasing where x'' is positive. This is where x is concave upward,

$0 < t < 2$

$\boxed{A}$

22. rate prop. to (# heard) $\cdot$ (# not heard)

$\dfrac{dp}{dt} = k \cdot p \cdot (N - p)$

$\boxed{B}$

23. $\frac{dy}{dx} = \frac{x^2}{y}$

$y\,dy = x^2\,dx$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C_1$

$y^2 = \frac{2}{3}x^3 + C_2$

$y(3) = -2$

$(-2)^2 = \frac{2}{3}(3)^3 + C_2$

$4 = 18 + C_2$

$-14 = C_2$

$y^2 = \frac{2}{3}x^3 - 14$

$y = -\sqrt{\frac{2}{3}x^3 - 14}$

$\boxed{E}$

24. Tan. Line:

$y - 1 = 4(x-2)$

$y = 4(x-2) + 1$

$f(1.9) \approx 4(1.9-2) + 1$

$= -.4 + 1$

$= .6$

$\boxed{B}$

25. Since f is continuous

$2c + d = 4 - 2c$

$f'(x) = \begin{cases} c, & x \le 2 \\ 2x - c, & x > 2 \end{cases}$

Since f is differentiable

$c = 4 - c$

$2c = 4$

$c = 2$

$4 + d = 4 - 4$

$d = -4$

$c + d = 2 - 4$

$= -2$

$\boxed{B}$

26. $y = \arctan(4x)$

$y' = \frac{4}{1 + (4x)^2}$

$y'(\frac{1}{4}) = \frac{4}{1 + (4 \cdot \frac{1}{4})^2}$

$= \frac{4}{1+1}$

$= 2$

$\boxed{A}$

27. $\frac{dy}{dx}$ must be zero when $x = -1$ and when $y = 0$

$\boxed{C}$

28. $\underline{f}$ $\qquad$ $\underline{g}$

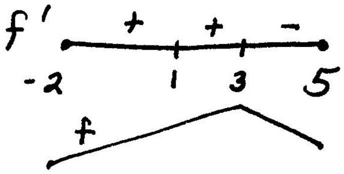
point$(6, 3)$ $\qquad$ point$(3, 6)$

$m = -2$ $\qquad$ $m = -\frac{1}{2}$

$\boxed{A}$

Part B

29. f'



$\boxed{B}$

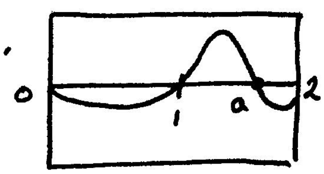
30. I $\lim_{x \to 2^-} f(x) =$ y-value at hole

II $\lim_{x \to 2^+} f(x) =$ y-value at horiz. seg

III $\lim_{x \to 2} f(x)$ does not exist (since I and II are different)

$\boxed{C}$

31.



graph of f'

$a = 1.691$

f is inc. where f' is pos.

$1 < x < 1.691$

$\boxed{B}$

32. $\int_{-5}^{5} f(x)dx =$

$\int_{-5}^{2} f(x)dx + \int_{2}^{5} f(x)dx =$

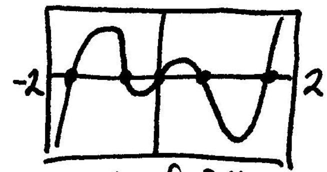
$\int_{-5}^{2} f(x)dx - \int_{5}^{2} f(x)dx =$

$-17 - (-4) =$

-13 $\qquad$ $\boxed{B}$

33. $f''(x) = x^2(-\sin x^2)2x + \cos x^2 \cdot 2x$

$= -2x^3 \sin x^2 + 2x \cos x^2$



graph of f''

f'' changes sign at 5 points

f has 5 inf. pts.

$\boxed{E}$ (You could also look for rel. ext. on the graph of f'.)

34. $G(4) = G(2) + \int_{2}^{4} f(x)dx$

$= -7 + \int_{2}^{4} f(x)dx$

$\boxed{E}$

35. $v(t) = 7 - (1.01)^{-t^2}$

$a(3) = v'(3) = .055$

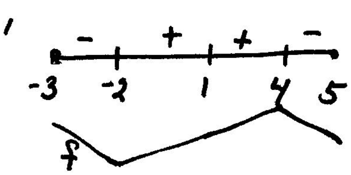
$\boxed{B}$

36. $A = \int_{1}^{2}(x^3 - 8x^2 + 18x - 5 - (x+5))dx + \int_{2}^{5}(x+5-(x^3-8x^2+18x-5))dx$

$= 11.833$

$\boxed{B}$

37. f'



$\boxed{C}$

38. $\int_{-4}^{-1} f'(x)dx =$

$f(x) \Big|_{-4}^{-1} =$

$f(-1) - f(-4) =$ $\qquad$ $\boxed{B}$

$-1.5 - .75 = -2.25$

39. Since v changes sign, x(t) has a min. between t=0 and t=1 and a max. between t=2 and t=4 (probably at t=3).

$\boxed{C}$

40. $x(3) = x(0) + \int_0^3 v(t)\,dt$

$= 2 + \int_0^3 \sqrt[3]{1+t^2}\,dt$

$= 6.512$

$\boxed{D}$

41. $\frac{dr}{dt} = -2$ cm/sec, $r = 3$ cm

$\frac{dS}{dt} = ?$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi \cdot 3(-2)$

$= -48\pi$ cm²/sec

$\boxed{C}$

42. If there is no c-value, MVT does not apply, f must not be differentiable at some point on the interval (-2, 2)

$\boxed{E}$

43. Since $f''(x) < 0$, $f'(x)$ must be decreasing. This means the slopes on f must be decreasing.

AROC on [2,3] > $f'(3)$ > AROC on [3,4]

AROC on [2,3] > 2 > AROC on [3,4]

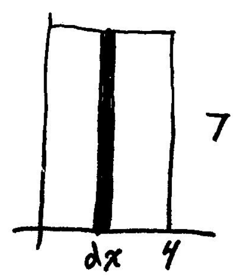
only $\boxed{A}$ works

44. $f_{avg} = \dfrac{\int_{-1}^{3} \frac{\cos x}{x^2+x+2}\,dx}{3+1}$

$= .183$

$\boxed{C}$

45.



area of rect. strip = 7dx sq. miles

Population living in rect. strip = $7dx \cdot f(x)$

City Pop $= \int_0^4 7f(x)\,dx$

$= 7\int_0^4 f(x)\,dx$

$\boxed{B}$