CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

Name _____

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\lim_{x\to\infty} \dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

2. $\displaystyle\int \frac{1}{x^2}\,dx =$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If $f(x) = (x-1)\left(x^2+2\right)^3$, then $f'(x) =$

(A) $6x\left(x^2+2\right)^2$ (B) $6x(x-1)\left(x^2+2\right)^2$ (C) $\left(x^2+2\right)^2\left(x^2+3x-1\right)$

(D) $\left(x^2+2\right)^2\left(7x^2-6x+2\right)$ (E) $-3(x-1)\left(x^2+2\right)^2$

4. $\displaystyle\int(\sin(2x)+\cos(2x))\,dx =$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$ (B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$ (C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$ (E) $-2\cos(2x)+2\sin(2x)+C$

5. $\lim_{x\to 0} \dfrac{5x^4+8x^2}{3x^4-16x^2}$ is

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) nonexistent

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

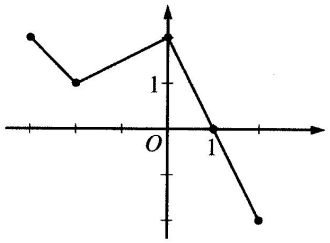
(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

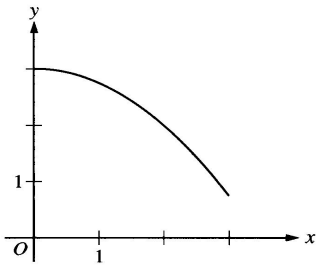
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



Graph of f

10. The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

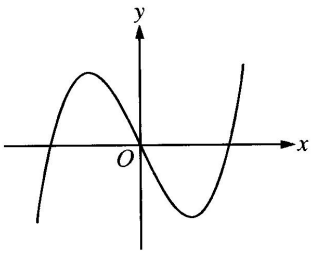
(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

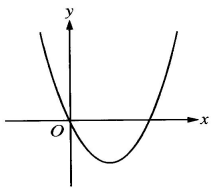
(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

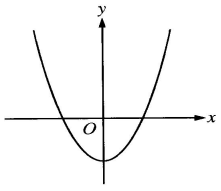
(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

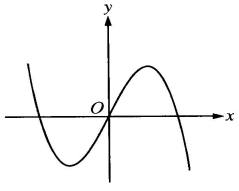


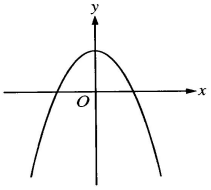
Graph of f

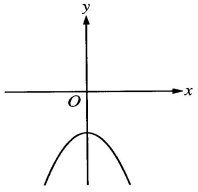
11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

(A) 

(B) 

(C) 

(D) 

(E) 

12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{\left(-2/x^2\right)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}\left(f(\ln x)\right) =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2x$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x + 2}{x}$

x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

15. $\displaystyle\int \frac{x}{x^2 - 4}\,dx =$

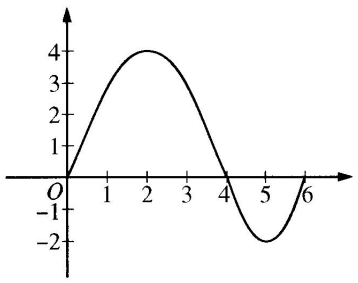
(A) $\dfrac{-1}{4(x^2-4)^2} + C$ (B) $\dfrac{1}{2(x^2-4)} + C$ (C) $\dfrac{1}{2}\ln|x^2 - 4| + C$

(D) $2\ln|x^2 - 4| + C$ (E) $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

16. If $\sin(xy) = x$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{1}{\cos(xy)}$ (B) $\dfrac{1}{x\cos(xy)}$ (C) $\dfrac{1 - \cos(xy)}{\cos(xy)}$

(D) $\dfrac{1 - y\cos(xy)}{x\cos(xy)}$ (E) $\dfrac{y(1 - \cos(xy))}{x}$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

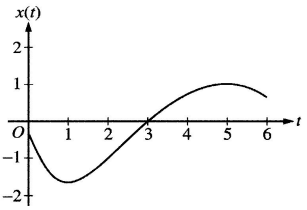
19. What are all horizontal asymptotes of the graph of $y = \dfrac{5 + 2^x}{1 - 2^x}$ in the xy-plane?

(A) $y = -1$ only (B) $y = 0$ only (C) $y = 5$ only

(D) $y = -1$ and $y = 0$ (E) $y = -1$ and $y = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x - 3)(x - 6)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, and 6



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$ (B) $1 < t < 5$ (C) $2 < t < 6$

(D) $3 < t < 5$ only (E) $1 < t < 2$ and $5 < t < 6$

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\dfrac{dp}{dt} = kp$ (B) $\dfrac{dp}{dt} = kp(N - p)$ (C) $\dfrac{dp}{dt} = kp(p - N)$

(D) $\dfrac{dp}{dt} = kt(N - t)$ (E) $\dfrac{dp}{dt} = kt(t - N)$

23. Which of the following is the solution to the differential equation $\dfrac{dy}{dx} = \dfrac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$ (B) $y = -2e^{-9+x^3/3}$ (C) $y = \sqrt{\dfrac{2x^3}{3}}$

(D) $y = \sqrt{\dfrac{2x^3}{3} - 14}$ (E) $y = -\sqrt{\dfrac{2x^3}{3} - 14}$

24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

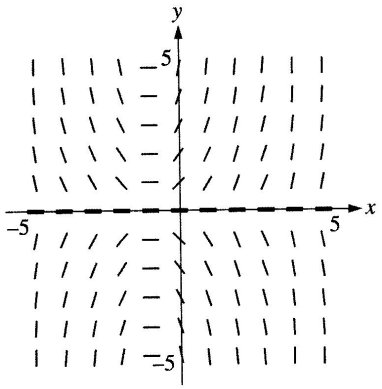
$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2



27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$ (B) $\frac{dy}{dx} = xy - y$ (C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$ (E) $\frac{dy}{dx} = (x + 1)^3$

28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

CALCULUS AB

SECTION I, Part B

Time—50 minutes

Number of questions—17

Name _____

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

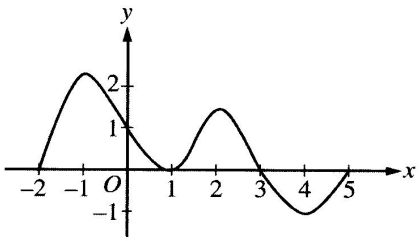
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

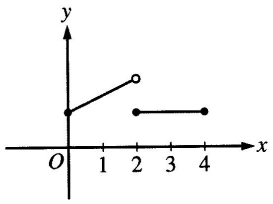


Graph of f'

29. The graph of f', the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only (B) $[-2, 3]$ (C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$ (E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$



Graph of f

30. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists. II. $\lim_{x \to 2^+} f(x)$ exists. III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

31. The first derivative of the function f is defined by $f'(x) = \sin\left(x^3 - x\right)$ for $0 \le x \le 2$. On what intervals is f increasing?

(A) $1 \le x \le 1.445$ only (B) $1 \le x \le 1.691$ (C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$ (E) $0 \le x \le 1$ and $1.691 \le x \le 2$

32. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

33. The derivative of the function f is given by $f'(x) = x^2 \cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

34. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$ (B) $-7 + f'(4)$ (C) $\int_{2}^{4} f(t)\,dt$

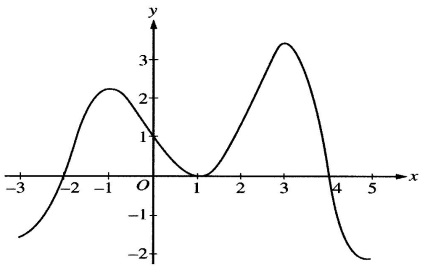
(D) $\int_{2}^{4} (-7 + f(t))\,dt$ (E) $-7 + \int_{2}^{4} f(t)\,dt$

35. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

36. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32



Graph of f'

37. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

(A) -2 only (B) 1 only (C) 4 only (D) -1 and 3 only (E) -2, 1, and 4

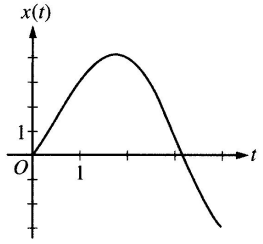
x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

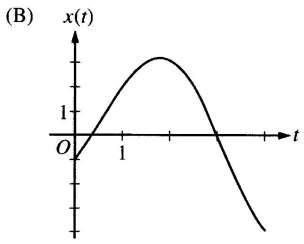
38. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

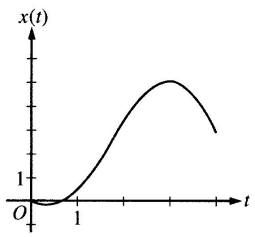
(A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

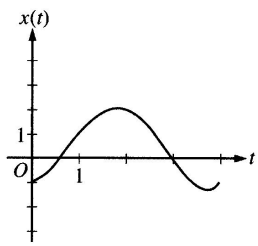
t	0	1	2	3	4
$v(t)$	−1	2	3	0	−4

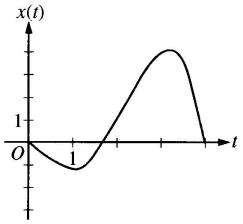
39. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

(A) 

(B) 

(C) 

(D) 

(E) 

40. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

41. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

42. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

43. The function f is continuous on the closed interval $[2, 4]$ and twice differentiable on the open interval $(2, 4)$. If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2, 4)$, which of the following could be a table of values for f ?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

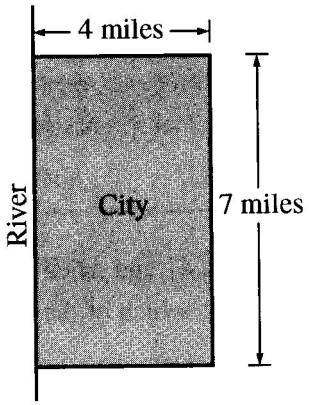
x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5

44. What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

(A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732



45. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)\,dx$ (B) $7\int_0^4 f(x)\,dx$ (C) $28\int_0^4 f(x)\,dx$ (D) $\int_0^7 f(x)\,dx$ (E) $4\int_0^7 f(x)\,dx$