2003 MULTIPLE CHOICE SOLUTIONS - NONCALCULATOR

1. $y = (x^3+1)^2$

$\frac{dy}{dx} = 2(x^3+1)(3x^2) = 6x^2(x^3+1)$

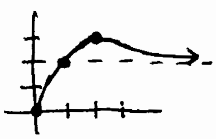
$\boxed{E}$

2. $\int_0^1 e^{-4x}\,dx = \frac{-e^{-4x}}{4}\Big|_0^1$

$= \frac{-e^{-4}}{4} + \frac{1}{4}$ OR $\frac{1}{4} - \frac{e^{-4}}{4}$

$\boxed{D}$

3. HOW ABOUT THIS GRAPH?



$f(0)=0$, $f(1)=2$

$f(2)=3$, $\lim_{x\to 2} f(x) = 3$

* $\lim_{x\to\infty} f(x) = 2$ (ONLY E MUST BE TRUE)

$\boxed{E}$

4. $y = \frac{2x+3}{3x+2}$

$y' = \frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2}$

$= \frac{6x+4-6x-9}{(3x+2)^2} = \frac{-5}{(3x+2)^2}$

$\boxed{D}$

5. $\int_0^{\frac{\pi}{4}} \sin x\,dx = -\cos x\Big|_0^{\frac{\pi}{4}}$

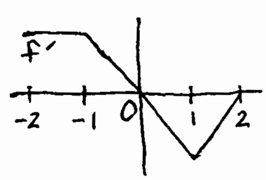
$= -\cos\frac{\pi}{4} + \cos 0 = -\frac{\sqrt{2}}{2} + 1$

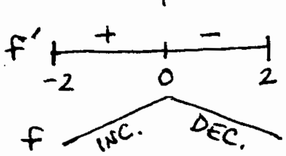
$\boxed{D}$

6. $\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} = \frac{1}{4}$

$\boxed{C}$

7.





$\boxed{B}$

NOTE: f MUST BE DIFFERENTIABLE AT $x=-1$ AND $x=1$ OR ELSE f' WOULD NOT EXIST AT THOSE VALUES OF x.

8. $\int x^2 \cos(x^3)\,dx =$

$\frac{1}{3}\int \cos(x^3)\cdot 3x^2\,dx = \frac{1}{3}\sin(x^3) + C$

$\boxed{B}$

9. $f(x) = \ln(x+4+e^{-3x})$

$f'(x) = \frac{1-3e^{-3x}}{x+4+e^{-3x}}$

$f'(0) = \frac{1-3}{0+4+1} = -\frac{2}{5}$

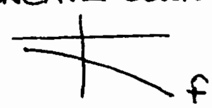
$\boxed{A}$

10. $f(x)$ IS NEGATIVE IMPLIES THE f GRAPH IS BELOW THE X-AXIS.

$f'(x)$ IS NEGATIVE IMPLIES THE f GRAPH IS DECREASING.

$f''(x)$ IS NEGATIVE IMPLIES THE f GRAPH IS CONCAVE DOWNWARD

$\boxed{B}$



11. FOR $u = 2x+1$,

$du = 2dx \rightarrow dx = \frac{1}{2}du$

AND $\int_0^2$ (x LIMITS) $\rightarrow \int_1^5$ (u LIMITS)

$\int_0^2 \sqrt{2x+1}\,dx = \frac{1}{2}\int_1^5 \sqrt{u}\,du$

$\boxed{C}$

12. $\frac{dV}{dt} = k\cdot\sqrt{V}$

RATE OF CHANGE OF V WITH RESPECT TO t — IS PROPORTIONAL TO THE SQUARE ROOT OF V

$\boxed{E}$

13. WHEN $x=a$, f IS CONTINUOUS BUT NOT DIFFERENTIABLE. (SHARP TURN)

$\boxed{A}$

14. $y = x^2\sin(2x)$ PRODUCT

$\frac{dy}{dx} = x^2\cos(2x)\cdot 2 + 2x\sin(2x)$

$= 2x\left(\sin(2x) + x\cos(2x)\right)$

$\boxed{E}$

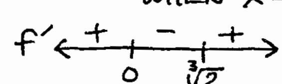
15. $f'(x) = x^2 - \frac{2}{x}$ UNDEFINED WHEN $x=0$

$x^2 - \frac{2}{x} = 0$

$x^3 - 2 = 0$

$x^3 = 2$

$x = \sqrt[3]{2}$



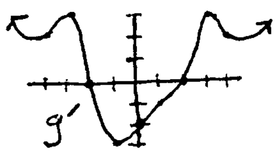
$\boxed{D}$

f IS DECREASING ON $(0, \sqrt[3]{2}]$

16. POINTS: $(1,7)$ ON f AND $(-2,-2)$ ON T.L.
SLOPE: $m = \dfrac{-2-7}{-2-1} = \dfrac{-9}{-3} = 3$
$f'(1) = 3$
$\boxed{C}$

17. $f(x) = 2xe^x$
$f'(x) = 2xe^x + 2e^x$
$f''(x) = 2xe^x + 2e^x + 2e^x$
$= 2e^x(x+2)$
$2e^x(x+2) < 0$ WHEN $x+2<0$
$x<-2$
(CONCAVE DOWN)
$\boxed{A}$

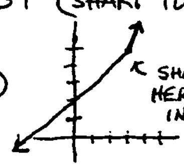
18. g IS DECREASING WHEN $g' \le 0$
(OR $g'<0$)
INTERVAL CAN BE CONSIDERED OPEN OR CLOSED.
$-2 \le x \le 2$
(SEE POSSIBLE GRAPH OF g' AT RIGHT)



$\boxed{A}$

19. SLOPE EQUATION: $y' = 2x+3$
$y = x^2+3x+C$
$2 = 1+3+C \rightarrow C=-2$
$y = x^2+3x-2$
$\boxed{D}$

20. $f(x) = \begin{cases} x+2 & \text{IF } x\le 3 \\ 4x-7 & \text{IF } x>3 \end{cases}$
$\lim_{x\to 3} f(x) = 5$ AND $f(3)=5$,
SO I AND II ARE TRUE
$f'(x) = \begin{cases} 1 & \text{IF } x<3 \\ 4 & \text{IF } x>3 \end{cases}$
$f'(3)$ DOES NOT EXIST (SHARP TURN)
III IS NOT TRUE
(SEE GRAPH AT RIGHT)



$\boxed{D}$

21. POINTS OF INFLECTION OCCUR WHEN f'' CHANGES SIGN. THAT HAPPENS AT $x=a$ AND AT $x=0$ (SEE GRAPH)
$\boxed{A}$

22. $f(1) = f(0) + \int_0^1 f'(x)\,dx$
$= 5 + \frac{1}{2}\cdot 1\cdot 6 = 5+3 = 8$
OR EQUATION OF LINE: $f'(x) = -6x+6$
$f(x) = -3x^2+6x+C$
$f(0) = 0+0+C = 5$
$\rightarrow f(x) = -3x^2+6x+5$
$f(1) = -3+6+5 = 8$
$\boxed{D}$

23. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) =$
$2x\sin(x^6)$
$\boxed{E}$

24. $f(x) = 4x^3-5x+3$
$f'(x) = 12x^2-5$
$f'(-1) = 12-5 = 7$ SLOPE $=7$
$f(-1) = -4+5+3 = 4$ POINT: $(-1,4)$
$y-4 = 7(x+1) \rightarrow y = 7x+11$
$\boxed{C}$

25. $x(t) = 2t^3-21t^2+72t-53$
$v(t) = 6t^2-42t+72$
AT REST: $v(t)=0$
$6t^2-42t+72=0$
$6(t^2-7t+12)=0$
$6(t-3)(t-4)=0$
$t=3$ AND $t=4$
$\boxed{E}$

26. $3y^2-2x^2 = 6-2xy$
$6yy'-4x = 0-2xy'-2y$
$6yy'+2xy' = 4x-2y$
$y'(6y+2x) = 4x-2y$
$y' = \dfrac{4x-2y}{6y+2x}$
$y'(3,2) = \dfrac{4(3)-2(2)}{6(2)+2(3)} = \dfrac{8}{18} = \dfrac{4}{9}$
$\boxed{B}$

27. $f(x) = x^3+x$ $\quad f'(x) = 3x^2+1$
$g(x) = f^{-1}(x)$ $\quad g'(2) = \dfrac{1}{f'(1)}$
$g(2) = 1$ $\quad = \dfrac{1}{4}$

OR

f	$g=f^{-1}$
$(1,2)$	$(2,1)$
$m=4$	$m=\frac{1}{4}$ (RECIPROCAL)
$f'(1)=4$	$g'(2)=\frac{1}{4}$

$\boxed{B}$

28. $g'(x)>0$ MEANS $g(x)$ IS INCREASING
$g''(x)>0$ MEANS $g(x)$ IS INCREASING AT AN INCREASING RATE (CONCAVE UP)
IF $g(4)=12$ AND $g(5)=18$, THEN $g(6)>24$

$\boxed{E}$

2003 MULTIPLE CHOICE SOLUTIONS

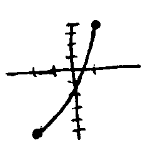
29. $V(t) = 3 + 4.1\cos(.9t)$
$a(t) = V'(t)$
$V'(t) = 1.633$
$\boxed{C}$ nDeriv$(3+4.1\cos(.9x), x, 4)$

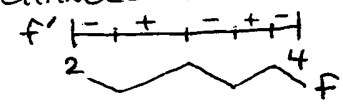
30. FROM THE GRAPH OF $f(x)$ AND THE GIVEN INFORMATION,
$\int_{-3}^{3} f(x)\,dx = -2+2-2 = -2$
$\int_{-3}^{3}(f(x)+1)\,dx = \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx$
$= -2 + x\Big|_{-3}^{3} = -2 + (3)-(-3) = 4$

OR THINK THAT ADDING 1 TO $f(x)$ BEFORE INTEGRATING ADDS THE AREA OF A 1 BY $\underset{(3--3)}{6}$ RECTANGLE TO $\int_{-3}^{3} f(x)\,dx$.
$-2+6=4$
$\boxed{C}$

31. $A = \pi r^2$ ← NOTE: THIS IS C
$\frac{dA}{dt} = \boxed{2\pi r}\frac{dr}{dt}$
SINCE $C = 2\pi r = 20\pi$ m. (or $r = 10$m.)
AND $\frac{dr}{dt} = .2$ m/sec.,
$\frac{dA}{dt} = 4\pi$ m^2/sec. AT THE GIVEN TIME.
$\boxed{C}$

32. IN GRAPH I, $\lim_{x\to 4} f(x)$ EXISTS
(it appears to be 3)
IN GRAPH II, $\lim_{x\to 4} f(x)$ EXISTS
(it appears to be 4)
$\lim_{x\to 4} f(x)$ DOES NOT EXIST FOR GRAPH III.
(LEFT AND RIGHT HAND LIMITS BOTH EXIST BUT ARE NOT EQUAL)
$\boxed{D}$

33. 
$f(c)=0$ AND $f(c)=3$ ON $(-2,1)$ MUST BE TRUE (IVT)
$f'(c)=3$ ON $(-2,1)$ MUST BE TRUE (MVT)
ON $[-2,1]$, $f(c) \ge f(x)$ FOR SOME C MUST BE TRUE (EXTREME VALUE THM)
$f'(c)=0$ DOES NOT HAVE TO BE TRUE.
$\boxed{B}$

34. GRAPH $f'(x) = \sin(x^2+1)$ ON THE INTERVAL $[2,4]$. YOU CAN SEE THAT f' CHANGES SIGNS 4 TIMES.


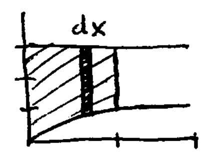
4 REL. EXTREMA
$\boxed{D}$

35. $r(t) = t^3 - 4t^2 + 6 \quad 0 \le t \le 8$
ALTITUDE IS DECREASING WHEN $r(t) < 0$
$r(t) = 0$ AT $t = 1.572$ AND $t = 3.514$
$r(t) < 0$ ON $(1.572, 3.514)$
CHANGE IN ALTITUDE $= \int_{1.572}^{3.514} r(t)\,dt$
$\boxed{A}$

36. $V_{avg} = \dfrac{\int_0^3 V(t)\,dt}{3-0} = 20.086$
$\boxed{A}$ fnInt$(e^x + xe^x, x, 0, 3) \div 3$

37. Temp. $= 350 + \int_0^5 (-110e^{-.4t})\,dt$
$\boxed{A}$ USE fnInt AGAIN.

38. A TRAPEZOIDAL SUM OVERAPPROXIMATES $\int_0^4 f(x)\,dx$ WHEN f IS CONCAVE UP.
A RIGHT RIEMANN SUM UNDERAPPROXIMATES $\int_0^4 f(x)\,dx$ WHEN f IS DECREASING.
$\boxed{A}$

39. $y = \tan^{-1}x$
$y = 3$ BOUNDARIES
$x = 1$


$V = \int_0^1 (3 - \tan^{-1}x)^2\,dx = 6.612$
$\boxed{B}$ YET ANOTHER USE OF fnInt.

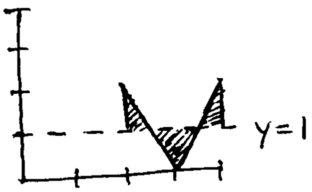
40. LET $y_1 = f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$
LET $y_2 = f''(x) =$ nDeriv(y_1, x, x)
THE GRAPH OF y_2 CROSSES THE X-AXIS AT $x = .473$
$f''(.473) = 0$ AND f'' CHANGES SIGNS AT THAT X-VALUE, THUS .473 IS THE X-COORDINATE OF THE INFLECTION POINT OF THE GRAPH OF f.
$\boxed{B}$

2003 MULTIPLE CHOICE SOLUTIONS

41. $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$ MEANS

THE AVG. VALUE OF $f(t)$ ON
(AVG. HEIGHT)
THE INTERVAL $[2,4]$ IS 1.
THUS, ON $[2,4]$, THE AREA BETWEEN f AND $y=1$ (ABOVE $y=1$) SHOULD EQUAL THE AREA BETWEEN f AND $y=1$ (BELOW $y=1$)

$\boxed{C}$



OR

$$\frac{1}{4-2}\int_2^4 f(t)\,dt = \frac{1}{2}\left(\tfrac{1}{2}\cdot 1\cdot 2+\tfrac{1}{2}\cdot 1\cdot 2\right)$$
$$= 1$$

42. $g(x) = x\cdot f(x)$

$g(2) = 2\cdot f(2) = 2\cdot 3 = 6$

POINT: $(2,6)$

$g'(x) = x\cdot f'(x) + f(x)\cdot 1$

$g'(2) = 2f'(2) + f(2)$

$= 2(-5) + 3 = -7$

$m = -7$

T.L: $y-6 = -7(x-2)$

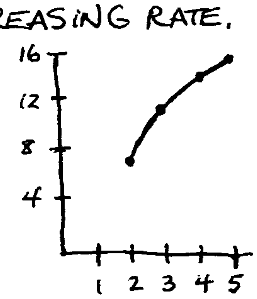
$\boxed{D}$

43. f HAS A POSITIVE FIRST DERIVATIVE IF f IS INCREASING.
f HAS A NEGATIVE SECOND DERIVATIVE IF f IS CONCAVE DOWN.
THUS, f IS INCREASING AT A DECREASING RATE.

$\boxed{B}$ FOR

x	f(x)
2	7
3	11
4	14
5	16

THE GRAPH OF f MUST BE INCREASING AND CONCAVE DOWN.



44. $V(2) = 2 + \int_1^2 \ln(1+2^t)\,dt$

$\boxed{E}$ $= 2 + 1.346 = 3.346$

USE fnInt ONCE AGAIN.

THOUGHT: $V(2) = V(1) +$ THE ACCUMULATED VELOCITY BETWEEN $t=1$ AND $t=2$.

45. $g(x) = \int_0^x \sin(t^2)\,dt$

FOR $-1 \le x \le 3$ IS DECREASING WHEN $g'(x) \le 0$ (COULD BE $g'(x) < 0$)

$g'(x) = \frac{d}{dx}\int_0^x \sin(t^2)\,dt$

$= \sin(x^2)$

$\sin(x^2) \le 0$ ON $[-1,3]$

FOR $1.772 \le x \le 2.507$

$\boxed{D}$